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NRL Report R-1669

THE SUBMERGED RECEPTION OF RADIO FREQUENCY SIGNALS

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December 2, 1940

NAVAL RESEARCH LABORATORY
Washington. D.C.

2 December 1940 NRL Report No. R-1669

NAVY DEPARTMENT

Report on

The Submerged Reception of

Radio Frequency Signals

NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

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Authorization: BuShips let. S67/46 (10-18-DR6) of 23 October 1940.

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APPENDICES

AUTHORIZATION

1. The general problem of investigating the submerged reception of low frequency radio signals was authorized in reference (a).

Reference: (a) BuShips ltr. S67/46(10-18-DR6) of 23 October 1940 to NRL (Problem B6-1).

INTRODUCTORY

2. The propagation of radio waves from a transmitting antenna in air to a receiving loop antenna submerged in sea water may be divided into the following parts:

Part I — The propagation of the radio wave from the transmitting antenna to the point at which the radio wave penetrates the sea water.

Part II — The refraction of the radio wave at the boundary between the air and the sea water.

Part III — The propagation of the radio waves in sea water.

Part IV — The voltage induced in a loop submerged in sea water.

Part V — The "Q" of a loop submerged in sea water.

Part VI — The r-f input voltage to a receiver.

3. A careful observation of the six parts given above reveals that

(a) Parts II and III do not enter in the ordinary radio communication problems.

(b) The voltage induced in a loop submerged in sea water may differ greatly from the value for the same loop in air.

(c) The "Q" of a loop will generally be less when submerged in sea water than in air.

4. This report is divided into Section I, "Theoretical Considerations," and Section II, "Experimental Considerations."

SECTION I

THEORETICAL CONSIDERATIONS

Part I – The propagation of radio waves from the transmitting antenna to the point at which the radio wave penetrates the sea water.

5. The propagation of low frequency radio waves along the surface of the earth (ground wave) has been studied by many investigators. Curves have been prepared giving the field strength as a function of the distance for one kilowatt of radiated power. From these curves the field strength of the ground wave at any distance from the transmitting antenna can be obtained. For distances less than 300 miles, for propagation over sea water and for frequencies below 100 kilocycles, the field strength is inversely proportional to the distance. The field strength at 100 miles for one kilowatt of radiated power is 1.9 millivolts per meter if the frequency is less than 100 killocycles and the propagation path is over sea water.

6. At the high frequencies the ground wave is attenuated more rapidly than at the low frequencies. The sky wave is refracted back to the earth with low absorption in the ionosphere so that the sky wave is generally much stronger than the ground wave. The <u>maximum</u> intensity of the sky wave is inversely proportional to the distance. The momentary intensity depends upon fading, polarization, scattering and absorption.

Part II – The refraction of the radio waves at the boundary between the air and the sea water.

7. The radio waves incident upon the boundary between the air and the sea water may be either reflected or refracted. The refracted wave penetrates into the second medium or the sea water. The ratio of the field strengths of the refracted wave to the incident wave (refraction coefficient) depends upon the angle of incidence, the dielectric constant and conductivity of both media, and the frequency of the radio wave. The refraction coefficient is given by the following formula.

$$R = \frac{2 \cos \phi \sin \theta}{\sin (\phi+\theta) \cos (\phi-\theta)} \qquad (1)$$

and

$$\frac{\sin \phi}{\sin \theta} = \sqrt{K - \frac{2\sigma}{f} i} = \frac{1}{\mu} \qquad (2)$$

where R is the coefficient of refraction.
ϕ is the angle of incidence (see Fig. 1).
θ is the angle of refraction (see Fig. 1).
K is the dielectric constant of sea water and is 80.
σ is the conductivity of sea water and is 3.6×10^{10} e.s.u.
f is the frequency in cycles per second.
$i = \sqrt{-1}$
μ is defined by Equation (2).
$a^2 = \sigma/f$

8. For sea water and for frequencies less than 10,000 kilocycles, Equation (2) is given to a high degree of accuracy by:

$$\frac{\sin\phi}{\sin\theta} = \sqrt{\frac{2\sigma}{f}}\, e^{-i\pi/4} = \sqrt{2}\, a\, e^{-i\pi/4} \tag{3}$$

9. In the case of the ground wave, the angle of incidence is approximately 90° (nearly grazing incidence). The angle of incidence is computed from the magnitude of the horizontal and vertical components of the electric field at the surface of the sea water. If the sea water were a perfect conductor, the horizontal component of the electric field would be zero, the angle of incidence would be 90°, and the radio wave would not penetrate the surface of the sea water. However, the sea water is not a perfect conductor; hence the horizontal component of the electric field is not zero, and thus the angle of incidence is less than 90°. Formulas for the horizontal and vertical components of the electric field at the surface of the earth have been derived by K. A. Norton* for the propagation of radio ground waves. The angle of incidence for the ground wave is given to a high degree of accuracy by Equation (4).

$$\cot\phi = \frac{E_H}{E_V} = \mu\sqrt{1 - \mu^2} \tag{4}$$

when E_H and E_V are the horizontal and vertical components of the electric field respectively.

10. Since ϕ is nearly 90° for the ground wave at the frequencies under consideration one may use the approximation $\cot\phi = \frac{\pi}{2} - \phi$, then Equation (4) becomes

$$\frac{\pi}{2} - \phi = \mu\sqrt{1 - \mu^2} \tag{5}$$

11. If one substitutes Equation (3) and (5) in Equation (1), one obtains for the coefficient of refraction for the ground wave between air and sea water.

$$R = 0.71\sqrt{\frac{f}{\sigma}} \tag{6}$$

12. It can be shown by numerical computation that the coefficient of refraction varies less than one per cent for angles of incidence between 0° and 89°. Thus the coefficient of refraction for the sky waves is not a function of the angle of incidence but only a function of the frequency.

13. The coefficient of refraction as computed from Equations (1) and (6) for the sky wave and ground wave respectively is plotted as a function of the frequency in Plate 1. It can be seen from the curves that the refraction coefficient is proportional to the square root of the

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*K. A. Norton, Proc. I.R.E., Vol. 25, page 1203 (1937).

frequency, is only a few thousandths at the lower frequencies and for the ground wave is about 50 per cent of that of the sky wave.

14. For the low frequencies, the sky wave is highly attenuated in the ionosphere while the attenuation of the ground wave is small. Thus the field strength of the ground wave for moderate distances is many times that of the sky wave for frequencies below 100 kilocycles, and Curve 1 in Plate 1 should be used to determine the refraction coefficient.

15. For the high frequencies the ground wave is very rapidly attenuated and the sky wave is refracted from the ionosphere with low absorption. Thus the field strength of the sky wave is many times that of the ground wave even for very short distances, and thus Curve 2 should be used.

Part III – The propagation of radio waves in sea water.

16. The general formula for the propagation of radio waves in sea water can be calculated from Maxwell's Field Equations, and is

$$E_a = E_o e^{\frac{-2\pi z}{c}\sqrt{\sigma f}} \cos\omega\left(t - \frac{x}{c} - \frac{za}{c}\right) \tag{7}$$

where E_a is the field strength at any distance z.

E_o is the field strength at z = o in the partially conducting medium.

z is the distance in cm.

c is the velocity of light and is 3×10^{10} cm per sec.

σ is the conductivity of the medium and is 3.6×10^{10} e.s.u. for sea water.

$a^2 = \sigma/f$.

f is the frequency in cycles per second.

Let

$$E_a = E_1 \cos\omega\left(t - \frac{x}{c} - \frac{za}{c}\right) \tag{8}$$

17. If one substitutes Equation (8) in Equation (7) and then the constants for sea water, one obtains

$$E_1/E_o = e^{-0.00398zf^{1/2}} \tag{9}$$

where z is the distance in meters and f is the frequency in cycles per second.

18. In Plate 2, the relative field strength E_1/E_o (as computed from Equation (9)) is plotted as a function of the depth z for a number of frequencies. Thus the relative field strength at any depth z and for any frequency can be read directly from these curves. It should be noted that the relative field strength at any depth z is not a function of the

angle of incidence of the radio wave and that the attenuation for any depth z increases rapidly with the increase in frequency.

Part IV – The voltage induced in a loop submerged in sea water.

19. It can be shown from Equation (2) that for all angles of incidence ϕ from the maximum value for the ground wave to 0^o or vertical incidence, the angle of refraction θ is nearly 0^o. Thus a vertically polarized wave in air becomes a horizontally polarized wave in sea water with its electric vector in the same vertical plane as direction of propagation in air.

20. Let us assume a rectangular loop immersed in sea water with the plane of the loop vertical and in the plane of the electric vector of the electromagnetic field. (See Fig. 2.) The electric vector is for all practical purposes parallel to the sides d of the loop and thus the voltage is induced in these two sides of the loop. Let E_1 be the field strength at a depth z or at the top edge of the loop (see Fig. 2). Then the induced voltage per turn in the loop in sea water is

$$V = d\,E_1 \left\{ \cos \omega \left(t - \frac{za}{c}\right) \left[1 - \cos\left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi}{c} \cdot b\sqrt{\sigma f}} \right] - \sin \omega \left(t - \frac{za}{c}\right) \sin\left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi}{c} \cdot b\sqrt{\sigma f}} \right\} \tag{10}$$

where V is the induced voltage in microvolts in the loop

E_1 is the field strength in microvolts per meter at the top of the loop

b is the height of the loop in meters

d is the width of the loop in meters

$a^2 = \frac{\sigma}{f}$

f is the frequency in cycles per second

$\omega = 2\pi f$

c is the velocity of light and is 3×10^8 meters per second.

The $\cos \omega (t - \frac{za}{c})$ term in Equation (10) is in phase with the electric field (compare with Equation (7)) and the $\sin \omega (t - \frac{za}{c})$ term is 90^o out of phase with the electric field.

21. If the one turn loop of Fig. 2 were in air, then the voltage induced in the loop would be

$$V_1 = bE \sin\left(\frac{\omega d}{c}\right) \sin \omega \left(t - \frac{x}{c}\right) \tag{11}$$

It should be noted in Equation (11) that the voltage induced in the loop is 90^o out of phase with respect to the electric field.

22. The cosine and sine terms of Equation (10) are plotted as a function of the frequency for a 2 feet square loop in Plate 3. It should be noted that the cosine term (Curve 1 in Plate 3) is larger than the

sine term. The square root of the sum of the squares of the sine and cosine terms in Equation (10) gives the magnitude of the voltage induced in a rectangular loop immersed in sea water and is

$$V = d\,E_1 \left\{ \left[1 - \cos\left(\frac{\omega b a}{c}\right) e^{-\frac{2\pi}{c} b \sqrt{\sigma f}} \right]^2 + \left[\sin\left(\frac{\omega b a}{c}\right) e^{-\frac{2\pi}{c} b \sqrt{\sigma f}} \right]^2 \right\}^{1/2} \tag{12}$$

23. The magnitude of the voltage induced per unit field strength in a 2-foot square loop immersed in sea water (as computed from Equation (12)) is plotted as a function of the frequency in Plate 4, Curve 1. Curve 1 corresponds to the square root of the sum of the squares of Curves 1 and 2 in Plate 3. Curve 2 in Plate 4 gives the induced voltage per unit field strength as a function of the frequency for the same one turn 2-foot square loop in air. If the field strength at the loop in air were the same as for the loop submerged in sea water, then the ratio of Curves 1 and 2 of Plate 4 gives the gain in the induced voltage when the loop is submerged in sea water. This gain is plotted as a function of the frequency in Plate 5. This gain is approximately inversely proportional to the frequency and is 1000 at 46 kilocycles. This gain when the loop is submerged in sea water partially compensates for the low coefficient of refraction at the surface of the sea water, the high attenuation of the radio waves with depth in sea water and the lower "Q" for a loop submerged in sea water; nevertheless, the loop under sea water functions at a tremendous disadvantage as compared to operation in air.

24. A study of Equations (10) and (12) and Plate 3 reveals that the induced voltage in a loop submerged in sea water approaches a maximum value as the frequency increases, and also that the cosine term of Equation (10) predominates even if the loop were only 0.5 foot on a side. This leads to the conclusion that the induced voltage in the loop submerged in sea water is due primarily to the difference in magnitude of the electric field at the top and bottom of the loop (see Fig. 2), and phase difference plays only a secondary part. For a loop in air, the induced voltage is equal to the phase difference of the electric vector at the two sides of the loop. Thus the operation of a loop in air and in sea water is not the same, and hence the general conclusions as to the design of loops for use in air may not necessarily hold for loops submerged in sea water.

25. In Plate 6 the induced voltage per unit field strength per turn is plotted as a function of the frequency for 0.5 foot, 1.0 foot, 2.0 feet and 1.0 meter square loops submerged in sea water. It should be noted that induced voltage increases rapidly as the size of the loop is increased, especially at the low frequencies. The total voltage induced in a loop is equal to the product of the number of turns and the induced voltage per turn. If the four square loops in Plate 6 were to be designed to have the same inductance, it would be found that the total induced voltage increases as the size of the loop is increased, with the

sine term. The square root of the sum of the squares of the sine and cosine terms in Equation (10) gives the magnitude of the voltage induced in a rectangular loop immersed in sea water and is

$$V = d\,E_1 \left\{ \left[1 - \cos\left(\frac{\omega b a}{c}\right) e^{-\frac{2\pi}{c} b\sqrt{\sigma f}} \right]^2 + \left[\sin\left(\frac{\omega b a}{c}\right) e^{-\frac{2\pi}{c} b\sqrt{\sigma f}} \right]^2 \right\}^{1/2} \tag{12}$$

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1 meter square loop being the most efficient.

26. A study of Equation (10) or (12) indicates that possibly a rectangular loop would be more efficient than a square loop. Let us consider a rectangular loop (see Fig. 2) for which the horizontal length d equals one meter and the vertical length b varies from 0.5 foot to 1 meter. The induced voltage per unit field strength per turn is given as a function of the frequency for these rectangular loops in Plate 7. It should be observed that the induced voltage approaches unity as the frequency increases. Again, if one assumes all the loops to have the same inductance, the total voltage induced is the greatest in the 1 meter square loop at the low frequencies, but at 1000 kilocycles the induced voltage in the 1 foot by 1 meter loop is 50 per cent greater than for the 1 meter square loop.

27. In practice the loop is tuned by approximately the same capacity at 30 kilocycles and 500 kilocycles, and hence the inductance of the loop must be inversely proportional to the square of the frequency. This fact is evident from Equation (13)

$$f^2 = \frac{1}{4 \pi^2 LC} \tag{13}$$

where L and C are the inductance of the loop and the tuning capacity, respectively. The inductance of the loop is proportional to the square of the number of turns N. If one substitutes N^2 for L in Equation (13) one finds that the number of turns is inversely proportional to the frequency. If one assumes a 400-turn 2-foot square loop at 20 kilocycles then the same size loop would have 40 turns at 200 kilocycles and 16 turns at 500 kilocycles. The total induced voltage per unit field strength is equal to the product of the induced voltage per unit field strength per turn and the number of turns. The total induced voltage per unit field strength is plotted in Plate 8 as a function of the frequency for a 2-foot square loop assuming 400 turns at 20 kilocycles. This curve was obtained by taking the product of the induced voltage per unit field strength per turn as read from Plate 6 and the number of turns. It should be noted that the voltage induced in the loop at 20 kilocycles is 100 times the field strength at the loop.

28. The product of the refraction coefficient (as read from Plate 1), the attenuation for any depth in sea water (as read from Plate 2) and the total induced voltage per unit field strength at the top of the loop (as read on Plate 8) give the total induced voltage in the loop per unit field strength above the surface of the water. This total induced voltage per unit field strength above the surface of the sea water is plotted as a function of the frequency for various depths of the 2-foot square loop in the sea water in Plates 9 and 10. Plates 9 and 10 are for the ground and sky wave respectively. A careful study of the curves in Plate 9 reveals the following facts.

(a) For the loop just submerged in sea water, the total voltage induced in the loop per unit field strength above the surface of the water decreases very slowly as the frequency increases.

(b) If the loop is submerged more than 5 feet, the total induced voltage decreases very rapidly as the frequency increases.

(c) The product of the "Q" of the loop and the total induced voltage gives the input voltage to the receiver.

(d) If the "Q" of the loop were 20 at all frequencies, then the input voltage to the receiver would be 0.05 of the field strength above the surface of the water for a 20-foot depth at 16 kilocycles; for a 10-foot depth at 61 kilocycles; 5-foot depth at 200 kilocycles and a 2-foot depth at 800 kilocycles.

Part V – The "Q" of a loop submerged in sea water.

29. In the design of loops to be submerged in sea water, it is necessary to maintain both the "Q" of the loop and the gain factor given in Plate 5. The product of the "Q" and the gain factor is a measure of the figure of merit of the loop. This figure of merit must be high in order to partially compensate for the low coefficient of refraction (Plate 1) and the high attenuation in sea water (Plate 2).

30. In the previous discussion it has been assumed that the electromagnetic field is not distorted appreciably by the presence of the loop. It is believed that if a single turn loop were separated from the sea water by a very thin layer of insulation, there would be little distortion of the electromagnetic field and the theory would be valid. In this case the sea water surrounding the receiving loop acts as a single turn closely coupled to the receiving loop. Since the "effective" conductivity of the sea water is inversely proportional to the frequency, the resistance of the sea water loop decreases with frequency. Thus the sea water loop lowers the effective "Q" of the receiving loop especially at the lower frequencies. The "Q" of such a receiving loop may be less than one.

31. Next let us assume that the receiving loop is enclosed in a very large volume of dielectric so that the field at the loop would be determined by the dielectric. Then the voltage induced in the loop per unit field strength would be about the same as in air and the gain factor in Plate 5 would be reduced to nearly unity. The "Q" of this loop would remain about the same as in air, but the figure of merit would be low because of the low gain factor.

32. Therefore, neither extreme of receiving loop design leads to a high figure of merit. Hence, the design for a loop of maximum figure of merit will be between the two extremes given in paragraphs 30 and 31.

Part VI – The r-f input voltage to the receiver.

33. If the dimensions of the receiving loop are small compared to a wave length, if the loop is tuned to resonance, and if the receiver is close to the loop, and the current at all points in the loop is the same, then the input voltage to a receiver is

$$V_r = E\ R\ A\ V\ N\ Q \tag{14}$$

where V_r is the input voltage to the receiver in microvolts.

E is the field strength in microvolts per meter just above the surface of the water.

R is the coefficient of refraction and is given in Plate 1.

A is the attenuation of the radio wave in sea water from the surface to the top of the loop. A can be obtained from Plate 2.

V is the voltage induced in the loop per unit field strength per turn. If the loop insulation does not distort the electromagnetic field near the loop appreciably, then V may be read from Plates 6 or 7 or computed from Equation (12).

N is the number of turns on the loop.

Q is the "Q" of the loop when submerged in sea water.

The use of Equation (14) is restricted at the higher frequencies because the "effective" dimensions of the loop may not be small compared to a wave length. The "effective" dimensions of the loop are "a" times the actual physical dimensions of the loop, where a $=\sqrt{\sigma/f}$.

34. The theory of receiving loops submerged in sea water will be illustrated by the following example. At 100 miles from NSS the field strength is 7200 microvolts per meter at 15.44 kilocycles. The top of the receiving loop is 20 feet below the surface of the sea water. The loop of 400 turns is square and is 2 feet on a side. What is the input voltage to the receiver?

(1) The field strength at the surface of the sea water is 7200 microvolts per meter.

(2) The refraction coefficient R is 0.00046 as read from Curve 1 of Plate 1. Thus the field strength just below the surface of the sea water is 7200 x 0.00046 or 3.3 microvolts per meter.

(3) The attenuation A for 20 feet of sea water at 15.44 kilocycles is 0.055 as read from Plate 2. The field strength 20 feet below the surface of the water is 3.3 x 0.055 or 0.18 microvolt per meter.

(4) The voltage induced in the loop per turn per unit field strength is 0.22 (see Curve 1 of Plate 4 or Curve 2 of Plate 6). The voltage induced in the loop per turn is 0.18 x 0.22 = 0.040 microvolt.

(5) The total voltage induced in the loop is 0.040 x 400 = 16 microvolts.

(6) The "Q" of the loop submerged in sea water is arbitrarily assumed to be 1.0 for the sake of numerical computation. The "Q" of the loop will depend upon the design. Thus the input voltage to the receiver is 16 x 1 = 16 microvolts.

35. If one computes the receiver input voltage from Equation (14) one obtains:

$$V_r = 7200 \times 0.00046 \times 0.055 \times 0.22 \times 400 \times 1.0 = 16 \text{ microvolts.}$$

SECTION II

EXPERIMENTAL CONSIDERATIONS

Measurements

36. In the theory, paragraph 29, it was pointed out that both the "Q" of the loop and the gain factor (defined in paragraph 23) must be maintained in order to have an efficient underwater loop. The figure of merit of a loop is equal to the product of the "Q" of the loop when submerged in sea water and the gain factor. The theoretical figure of merit is equal to the product of the "Q" in air and the theoretical gain factor. The theoretical gain factor for a 2-foot square loop is given in Plate 5. In paragraphs 30 and 31, it was shown that if one maintained either the "Q" in air or the theoretical gain factor, that figure of merit of the loop would be low. Thus, the loop must be a compromise design in order to have a high figure of merit. For the best compromise design, the loop must be surrounded by sufficient insulation so that the product of the "Q" of the loop when submerged in sea water and the gain factor will give the maximum figure of merit. Hence, a number of loops were designed and tested.

37. A block diagram of the experimental arrangement is shown in Figure 3. The galvanized iron tank, 2.5 feet in diameter and 4 feet high, was filled with sea water. A small transmitting loop antenna was mounted 3.5 feet above the surface of the water with plane of the loop vertical. The receiving loop was connected to the input terminals of the receiver by a short section of concentric transmission line and the system was tuned to resonance by a variable capacitor.

38. The procedure for making the measurements was as follows: The receiving loop was set at some known distance, either above or below the surface of the water, with the transmitting and receiving loops in the same plane. The current in the transmitting loop was noted and kept constant. The receiving loop was tuned to resonance by the variable tuning capacitor and the reading on the receiver output meter was noted. A signal generator was then connected to the input terminals of the receiver, and the signal generator voltage was adjusted to give the same output meter reading. The signal generator voltage is then equal to the receiver input voltage from the receiving loop or the product of the "Q" and the voltage induced in the loop. The measurements were repeated for the center of the loop at various distances above and below the surface of the sea water. The "Q" was measured at the receiver end of the transmission line for the loop in air and at various depths in sea water. The measurements that were made on the four loops are tabulated in Tables 1 to 4, inclusive.

Results

39. In the theory, the transmission path was assumed to be at least several miles, and hence, the induced voltage would be independent of the height of the receiving loop above the sea water for heights of

only a few feet. However, in the experimental arrangement, the transmission path was very short, and both electromagnetic field and the induced voltage in the receiving loop would be functions of the distance between the two loops. When the loop was submerged in the sea water, end or edge effects were to be expected because of the small physical dimensions of the tank.

40. The relative receiver input voltage (the product of induced voltage and the "Q" of the loop) is plotted as a function of the depth of the loop in sea water in Plates 11 to 14, inclusive, for the four loops. The depth measurements were from the surface of the water to the center of the loop. Negative depths indicate that the loop was above the surface of the water. The relative receiver input voltage is not constant above the surface of the water because of the proximity of the transmitting loop. It should be noted in Plates 11 to 14, inclusive, that the relative receiver input voltage does not drop abruptly when the loop is submerged in sea water. Thus, the receiver input voltage is approximately the same when the loop is above or just below the surface of the sea water. The input voltages to the receiver per unit field strength above the surface of the water for the receiving loop, both above and just below the surface of the sea water, are tabulated in Tables 1 to 4, inclusive. The values for the loop submerged in sea water were computed from Equation (14) with both A and E unity. A is unity because the top of the loop was assumed to be at the surface of the water. For the loop in air, the receiver input voltage was computed from Equation (15) with E assumed to be unity,

$$V_r = \frac{2\pi}{\lambda} B N E Q \qquad (15)$$

where λ is the wave length of the radio wave in meters, B is the area of the loop in square meters, N is the number of turns, Q is the "Q" of the loop in air, E is the field strength at the loop in microvolts per meter. V_r is the input voltage to the receiver in microvolts. V_r is approximately the same for each loop in air as in sea water. This is in agreement with the curves in Plates 11 to 14, inclusive, because there is no abrupt change in V_r as the loop is submerged in sea water. Therefore, both the induced voltage per unit field strength per turn in sea water and the gain factor (see paragraph 23) approach very closely to the theoretical values for these receiving loops, and hence, the electromagnetic field was not appreciably distorted by the presence of the loop, and V in Equation (14) can be computed from Equation (12) or read from Plates 6 or 7. Thus, loops can be designed with a figure of merit that will approach the theoretical value.

41. The "Q" of the receiving loops is plotted as a function of depth in sea water (measured to the center of the loop) in Plates 15 to 13, inclusive. Negative depths indicate that the center of the loop is above the surface of the water. If the loop is totally submerged, the "Q" is independent of the depth in all cases in Plates 15 to 18, inclusive. If the thickness of the insulation is the same, the percentage decrease in the "Q" when the loop is submerged in sea water increases as the diameter

of the loop increases (compare Plates 15 and 17). Thus, as the diameter of the loop increases, the thickness of the insulation must also increase if the percentage loss in "Q" when the loop is submerged in sea water is to remain the same.

42. The thickness of the insulation on loops A and B is the same (see Tables 1 and 2) except that for loop B the center of the doughnut is filled with insulating material. A comparison of Tables 1 and 2, Plates 11 and 12, and Plates 15 and 16 shows that the "Q", the gain factor and the figure of merit for the two loops are about the same. Hence, it may be possible to design loops that require more insulating material than in loop B and still have a loop with a high figure of merit.

43. The Naval Research Laboratory has made a series of measurements on submarine loop antennas of the clearing line type in the frequency range 300 to 1500 kilocycles. It was observed that the resistance of the loop increased many fold when the loop was submerged in sea water, and the resistance for a submerged loop decreased as the frequency was decreased. This is in agreement with paragraph 30. These experiments lead to the conclusion that the "Q" of a loop at the low frequencies may be decreased several hundred times when the loop is submerged in sea water.

44. In Plates 19 to 22, inclusive, the relative induced voltage in the loop is plotted as a function of the depth of the center of the loop in sea water. For the loop in air, in the experimental set-up, the induced loop voltage decreases as the loop approaches the surface of the water. This proves that the electromagnetic field above the surface of the water is not uniform. If the field above the surface of the water were uniform, then the induced loop voltage curve would be the same as the attenuation curves of radio waves in sea water. The curves for the induced loop voltage when the loop is submerged in sea water is equal to the product of the curve for the attenuation of radio waves in sea water and the curve for the attenuation of the radio waves in air when the tank is removed.

45. The curve for the attenuation in air when the sea water was removed was obtained in Plate 21 at 60 kilocycles by extending the attenuation curve in air PO to M. For a depth of 40 inches, the attenuation of the radio wave in air would be 0.042 as read from the curve OM while the total attenuation in sea water is 0.0143. Thus the attenuation in 40 inches of sea water is 0.0143/0.042 = 0.34. The attenuation of 40 inches of sea water at 60 kilocycles as read on Plate 2 is 0.37. Similar computations from Plate 22 give for the attenuation of radio waves in 40 inches of sea water 0.3 and 0.11 at 300 and 450 kilocycles, respectively. The attenuation of 40 inches of sea water as read from Plate 2 is 0.13 and 0.072 at 300 and 450 kilocycles, respectively. The agreement is as good as can be expected when one considers the assumptions that were made in obtaining the attenuation of the radio wave in 40 inches of air when the tank (sea water) was removed.

SUMMARY OF THE THEORY AND EXPERIMENTAL WORK

46. The receiver input voltage when the receiving loop is submerged in sea water is equal to the product of the following terms.

(a) The field strength in the air directly above the receiving loop.

(b) The refraction coefficient at the boundary between the air and sea water. The refraction coefficient is the ratio of the field strength just below the surface of the sea water to the field strength in air and is given as a function of the frequency in Plate 1. The refraction coefficient increases with the frequency and at 5700 kilocycles it is 20 times the value at 15 kilocycles.

(c) The attenuation of radio waves in sea water. The attenuation of radio waves in sea water increases rapidly with the frequency. (See Plate 2.)

(d) The induced voltage per turn per unit field strength at the top of the loop. This can be read from Plate 6 or 7 or computed from Equation (12) if the insulation surrounding the loop does not distort the electromagnetic field appreciably. The agreement between the experiment and the theory proves that the electromagnetic field was not distorted appreciably by the presence of the receiving loop. In fact the "Q" of the loop submerged in sea water may also be relatively high and approach the value for the loop in air. It was found that 0.345 inch of insulation on a loop 4 inches in diameter was sufficient to give a good "Q" at 60 kilocycles. If a small volume of dielectric material were placed in sea water, the electromagnetic field in the region of the dielectric material would be changed only slightly from the original state. Thus it is believed if a loop were enclosed in a dielectric medium of approximately the same dimensions as the loop and very small compared to a wave length that the induced loop voltage could still be computed from Equation (12) or read from Plates 6 and 7. For the same loop inductance, the total induced voltage increases as the diameter of the loop increases from 0.5 foot to 1.0 meter, especially at the lower frequencies.

(e) The number of turns in the loop.

(f) The "Q" of the loop when submerged in sea water. The thickness of the insulation must increase as the diameter of the loop increases in order to maintain the same "Q."

The receiver input voltage per unit field strength above the surface of the sea water is approximately constant if the loop is just below the surface of the water for frequencies from 10 to 500 kilocycles. For relatively strong signals above the surface of the sea water, the theory indicates that the signals could be received when the receiving loop is submerged 20 feet or more at the low frequencies and when the receiving loop is just submerged at the high frequencies.

47. The ratio of the "antenna effect" voltage pick-up to the loop pick-up voltage is many times lower when the loop is submerged in sea water, and thus the electrostatic shielding of the loop is not so essential. An electrostatic shield may be undesirable expecially if the shield is not insulated from the sea water.

48. Both the static and the desired signal are attenuated the same amount in the sea water; thus the ratio of the desired signal to static level remains the same at the input terminals of the receiver for the loop in air or submerged in sea water.

49. The general theory indicates that the operation of loops in air and submerged in sea water differs greatly; e.g., the vertical electric vector in air becomes for all practical purposes a horizontal electric vector in sea water. However, a loop antenna submerged in sea water can be used for direction finding purposes.

APPENDIX A

Induced Voltage in a Rectangular Loop Submerged in Sea Water

1. The top side of a rectangular loop (see Fig. 2) is submerged to a depth z in sea water. The field strength just below the surface of the sea water is E_o, the angle of refraction θ' is less than one degree for all angles of incidence, and thus the electric vector is for all practical purposes in the horizontal plane. Then the electric field at a depth z or at the top side of the loop is

$$E_a = E_o \cos \omega \left(t - \frac{x}{c} - \frac{za}{c}\right) e^{-\frac{2\pi}{c} z\sqrt{\sigma f}} \tag{16}$$

2. The electric field at a depth z_1 or at the bottom side of the loop is

$$E_b = E_o \cos \omega \left(t - \frac{x}{c} - \frac{z_1 a}{c}\right) e^{-\frac{2\pi}{c} z_1\sqrt{\sigma f}} \tag{17}$$

3. The induced voltage per turn in the loop is

$$V = \int_0^d (E_a - E_b)\, dx \tag{18}$$

4. It can be seen from Fig. 2 that

$$z_1 = z + b \tag{19}$$

5. From Equations (16), (17) and (19), one obtains for $(E_a - E_b)$

$$E_a - E_b = E_o e^{-\frac{2\pi}{c} z\sqrt{\sigma f}} \left\{ \cos \omega \left(t - \frac{x}{c} - \frac{za}{c}\right) \left[1 - \cos \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}}\right] - \sin \omega \left(t - \frac{x}{c} - \frac{za}{c}\right) \sin \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}} \right\} \tag{20}$$

6. If one substitutes Eq. (20) in Equation (18) and then performs the integration, one obtains for the induced voltage per turn in a rectangular loop

$$V = dE_o e^{-\frac{2\pi}{c} z\sqrt{\sigma f}} \left\{ \cos \omega \left(t - \frac{za}{c}\right) \left[1 - \cos \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}}\right] - \sin \omega \left(t - \frac{za}{c}\right) \sin \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}} \right\} \tag{21}$$

7. From Equation (16) the magnitude of the electric field E_a or the field strength at a depth z is

$$E_1 = E_o e^{\frac{2\pi}{c} z\sqrt{\sigma f}} \text{ or } E_1 = \left| E_a \right| \tag{22}$$

8. From Equations (21) and (22), one has

$$V = dE_1 \left\{ \cos \omega \left(t - \frac{za}{c}\right) \left[1 - \cos \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}}\right] - \sin \omega \left(t - \frac{za}{c}\right) \sin \left(\frac{\omega ba}{c}\right) e^{-\frac{2\pi b}{c}\sqrt{\sigma f}} \right\} \tag{23}$$

where V is the induced voltage per turn in a rectangular loop.

E_1 is the field strength at the top side of the loop.

b and d are the sides of the rectangular loop.

σ is the conductivity of sea water.

f is the frequency.

c is the velocity of light

$a^2 = \sigma/f$

$\omega = 2\pi f$

Table 1

Loop A

Inductance	3.805 m h
Inside diameter of winding	4.0"
Outside diameter of winding	4.7"
Cross section of winding	0.25" by 0.35"
Type of wire	20/38 Litz
Number of turns	139
Number of turns per layer	10
Number of layers	14
Thickness of insulation (superla wax)	0.345"

Frequency Kcs.	60	60	120	120
Loop in	Air	Sea Water	Air	Sea Water
Refraction coefficient	--	0.00182	--	0.0026
Induced loop voltage per turn	0.0000102	0.011	0.0000204	0.015
"Q" of loop	119	100	50	40
Receiver input voltage per unit field strength above surface of water.	0.17	0.27	0.14	0.23

Table 2

Loop B

Inductance	3.9 mh
Inside diameter of winding	4.0"
Outside diameter of winding	4.7"
Cross section of winding	0.25" by 0.35"
Type of wire	20/38 Litz
Number of turns	140
Number of turns per layer	10
Number of layers	14
Thickness of insulation (superla wax)	0.345"

Note: Center of doughnut was also filled with wax.

Frequency Kcs.	60	60	108	108
Loop in	Air	Sea Water	Air	Sea Water
Refraction coefficient	--	0.00182	--	0.00243
Induced loop voltage per turn	0.0000102	0.011	.000019	0.015
"Q" of loop	143	124	50	40
Receiver input voltage per unit field strength above surface of water.	0.20	0.34	0.13	0.20

Table 3

Loop C

Inductance	3.76 mh
Inside diameter of winding	8.0"
Outside diameter of winding	8.5"
Cross section of winding	0.205" by 0.25"
Type of wire	20/38 Litz
Number of turns	88
Number of turns per layer	8
Number of layers	11
Thickness of insulation (superla wax)	0.345"

Frequency Kcs.	60	60	108	108
Loop in	Air	Sea Water	Air	Sea Water
Refraction coefficient	--	0.00182	--	0.00243
Induced loop voltage per turn	0.000041	0.042	0.000074	0.055
"Q" of loop	136	89	58	32
Receiver input voltage per unit field strength above surface of water.	0.49	0.58	0.38	0.38

Table 4

Loop D

Inductance	142 mh
Inside diameter of winding	8.0"
Outside diameter of winding	8.1"
Cross section of winding	0.05" by 0.20"
Type of wire	20/38 Litz
Number of turns	16
Number of turns per layer	8
Number of layers	2
Thickness of insulation (superla wax)	0.345"

Frequency Kcs.	300	300	450	450
Loop in	Air	Sea Water	Air	Sea Water
Refraction coefficient	--	0.0041	--	0.005
Induced loop voltage per turn	0.00021	0.083	0.00031	0.11
"Q" of loop	95	45	65	23
Receiver input voltage per unit field strength above surface of water.	0.32	0.25	0.32	0.21

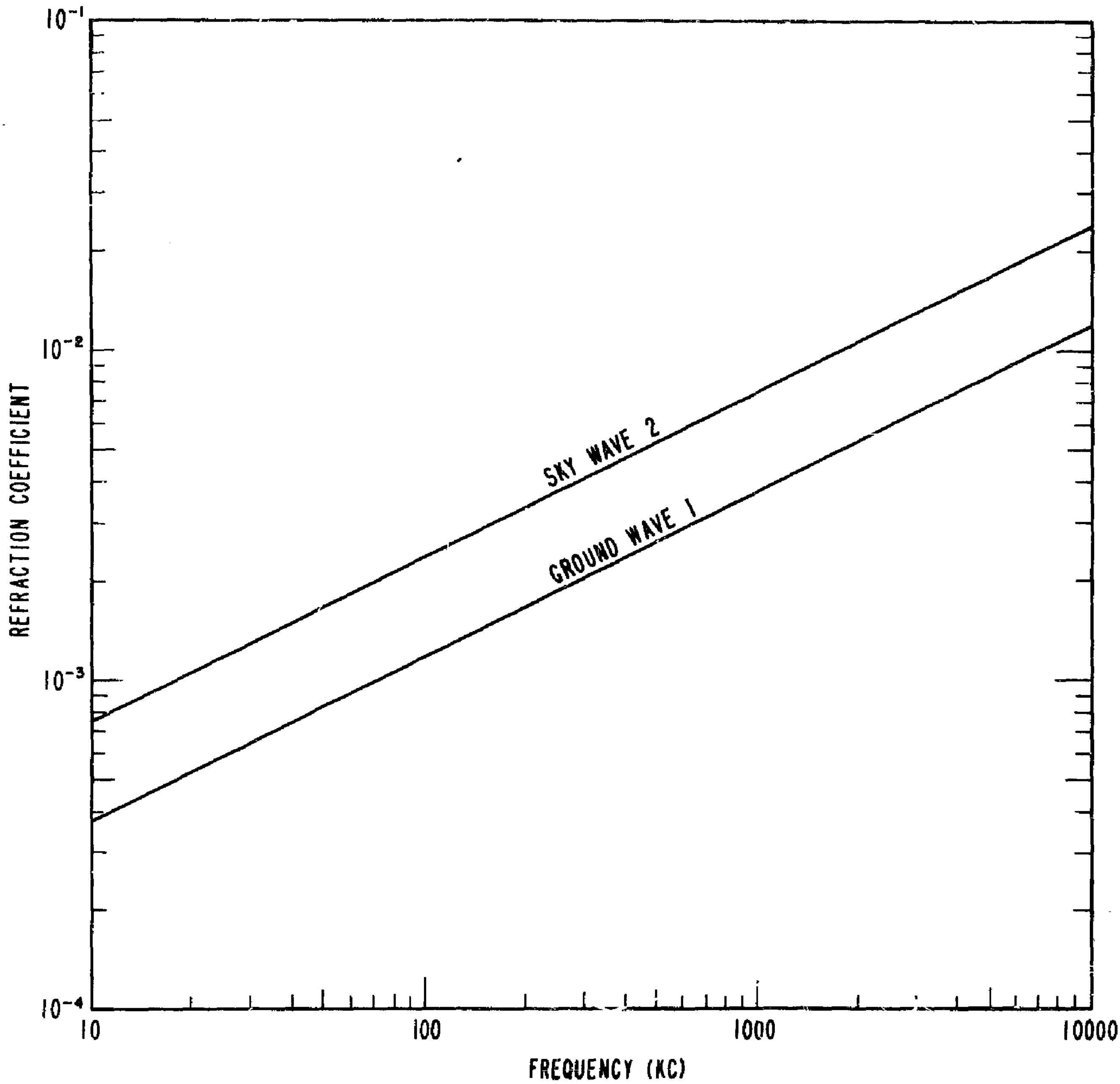

Plate 1 - Refraction coefficient of sea water as function of frequency of radio wave

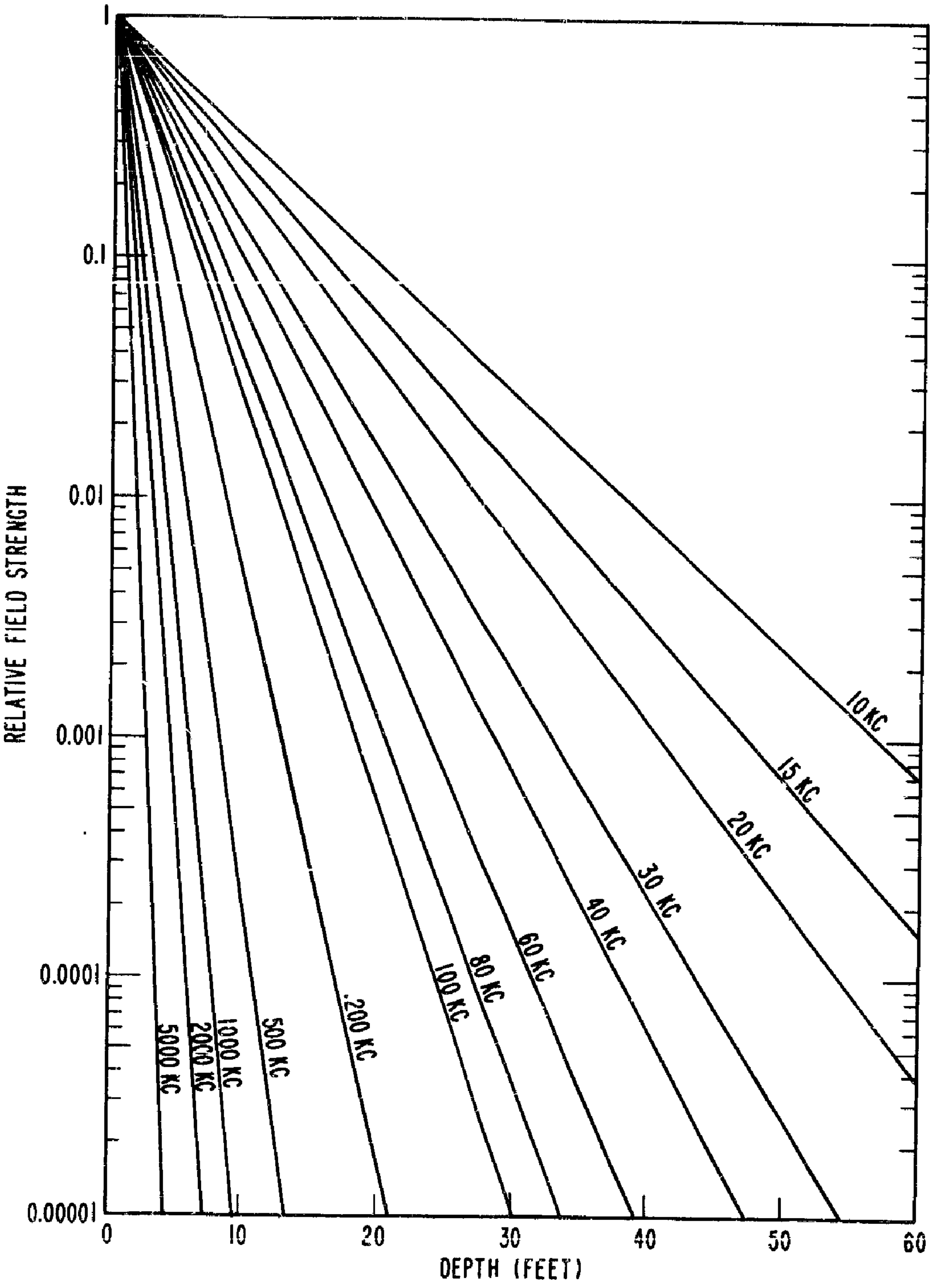

Plate 2 - Relative field strength as function of depth in sea water for various frequencies

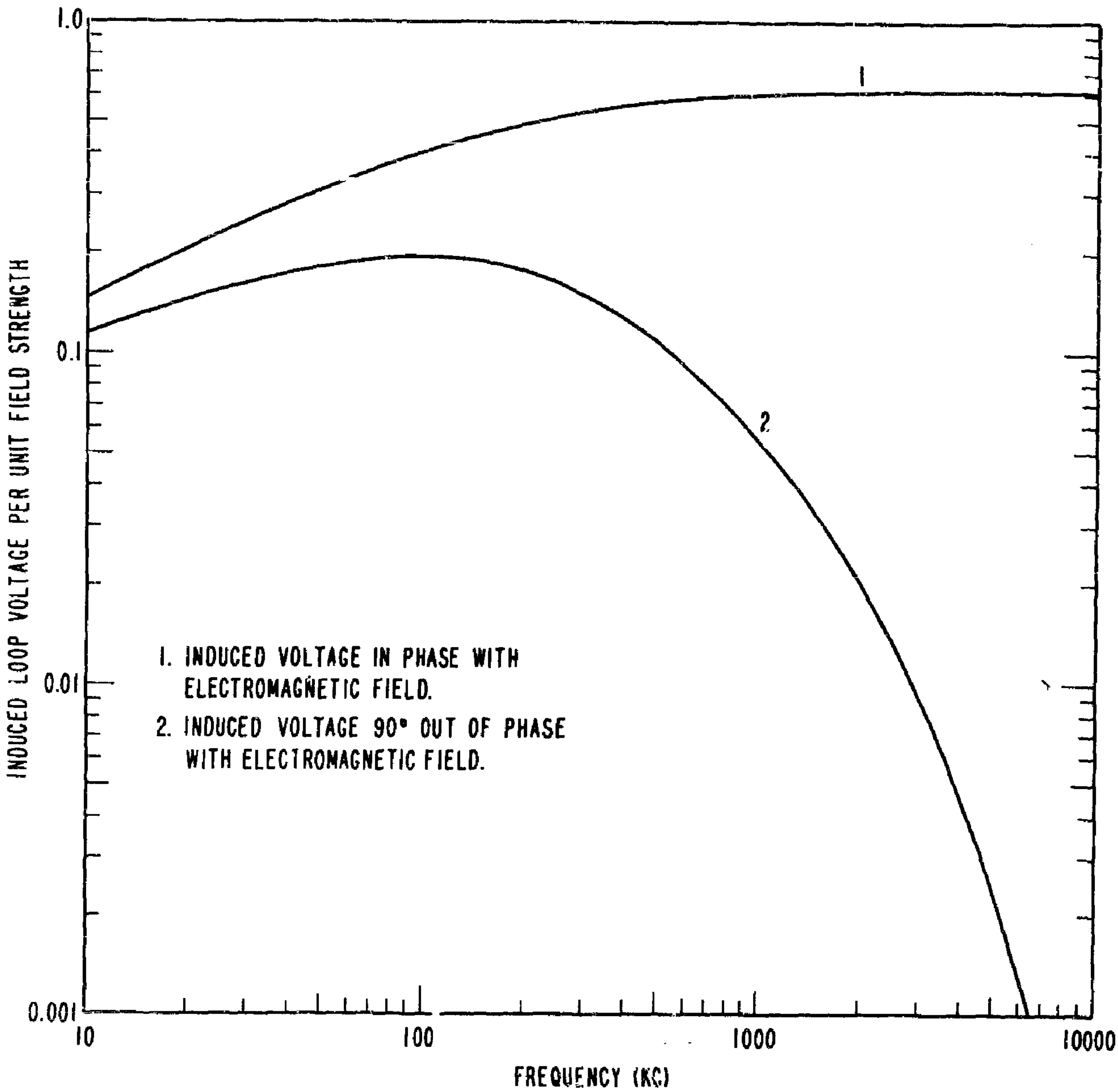

Plate 3 - Inductance voltage per unit field strength at top of one turn 2 ft square loop as function of frequency when loop is submerged in sea water

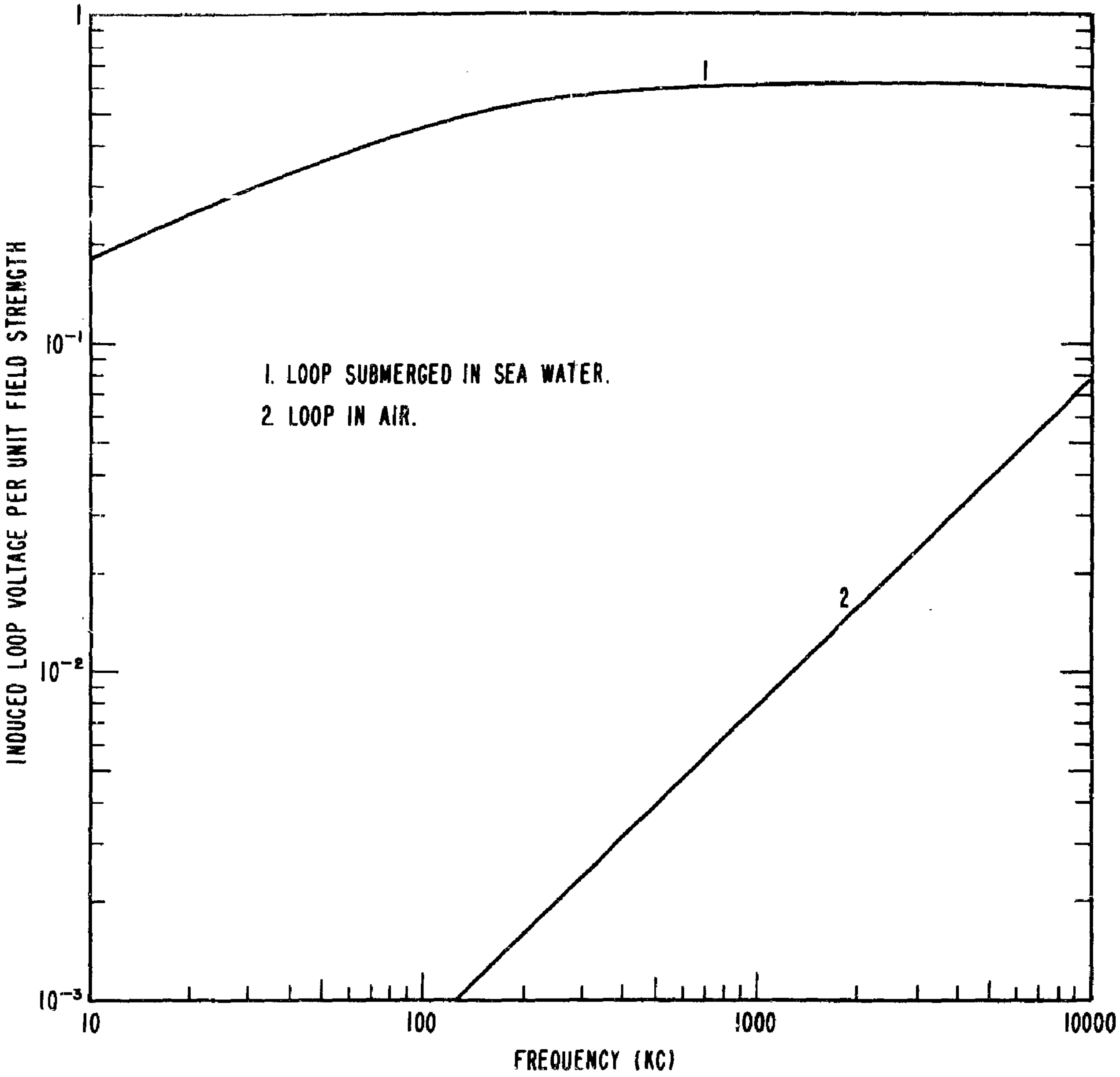

Plate 4 - Induced voltage per unit field strength as a function of frequency for one turn 2 ft square loop

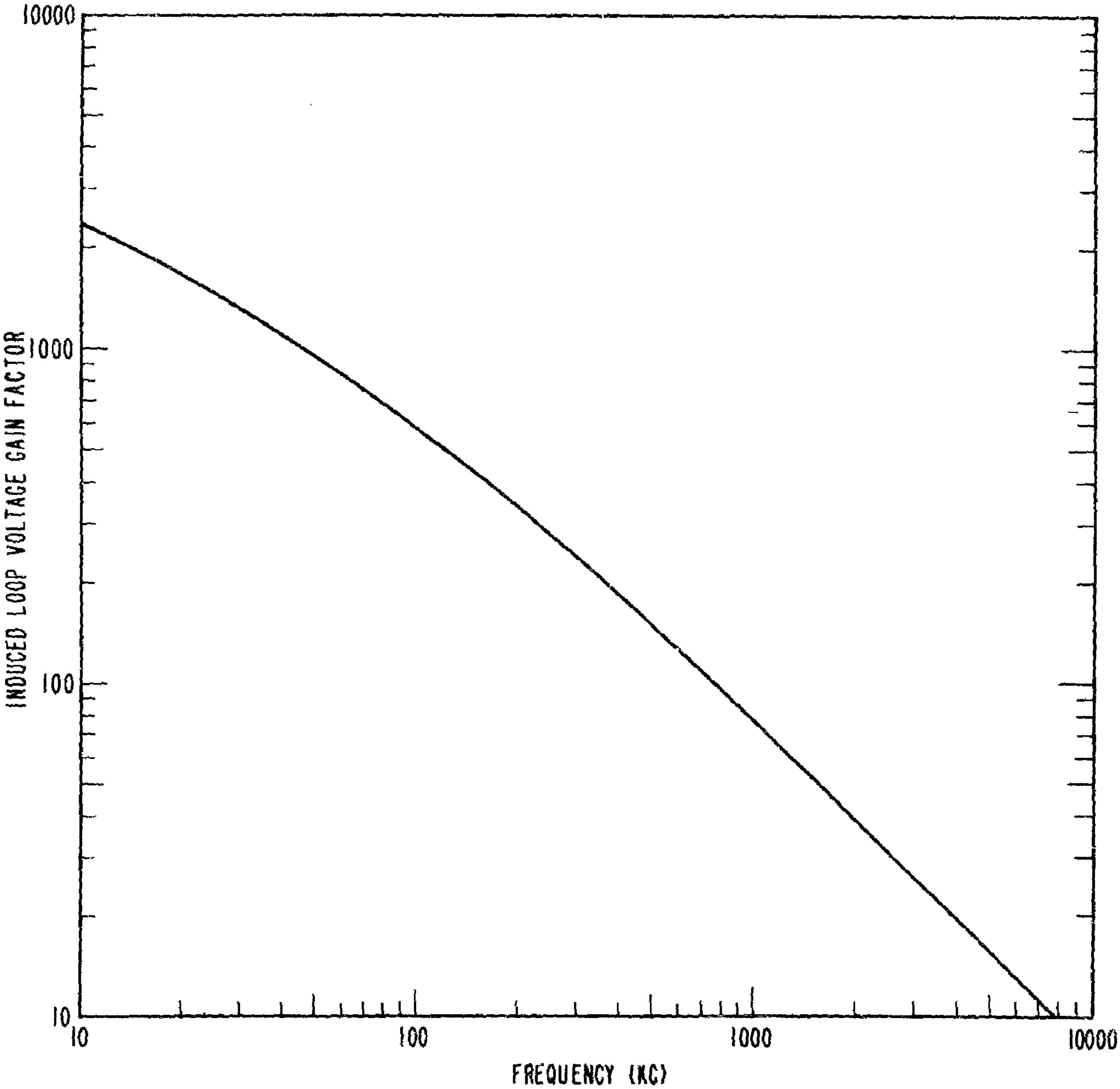

Plate 5 - Induced loop voltage gain factor as function of frequency for one turn 2 ft square loop

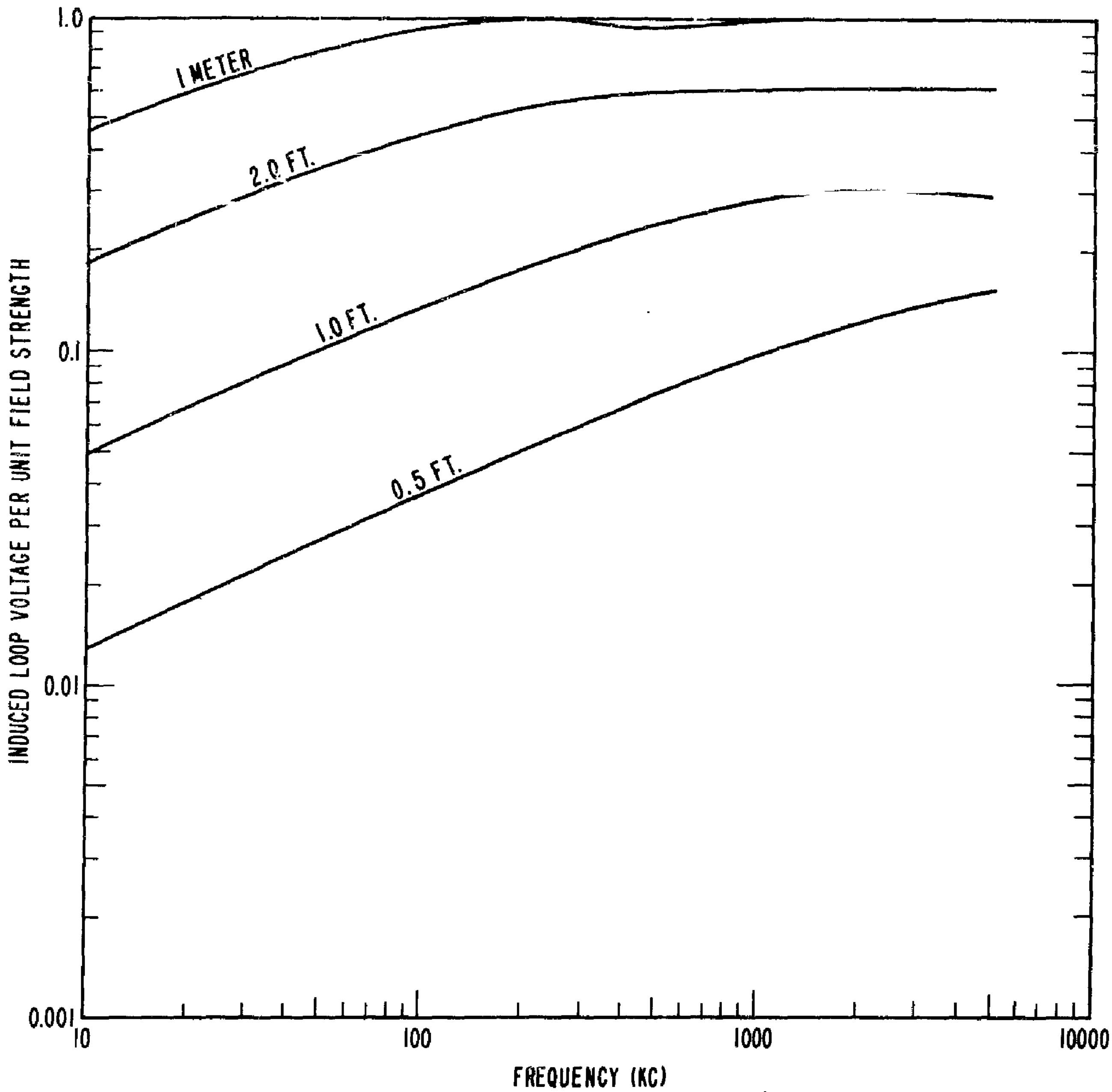

Plate 6 - Induced loop voltage per turn per unit field strength at the top of square loop as function of frequency for various sizes of loops

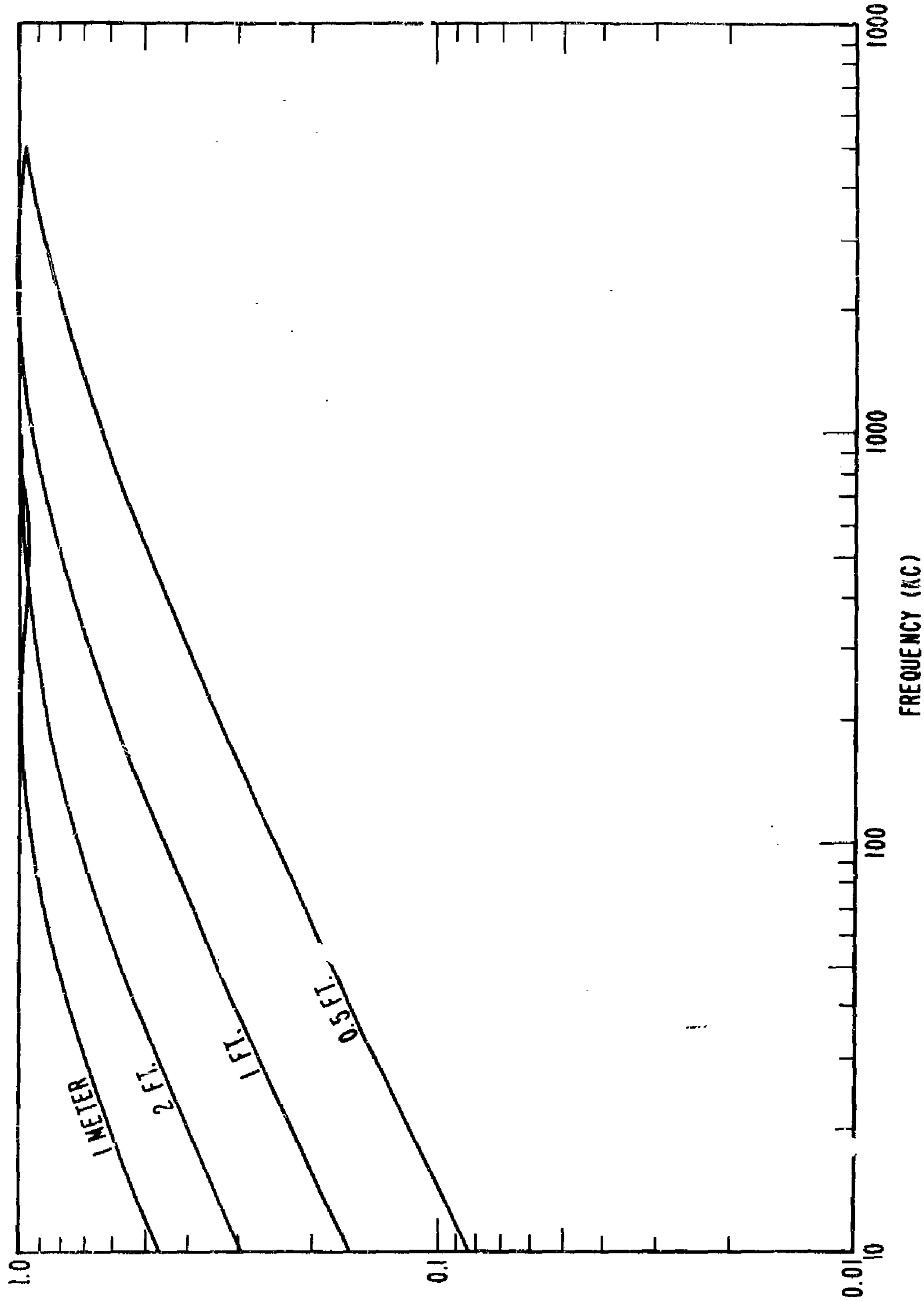

Plate 7 - Induced loop voltage per turn per unit field strength at the top of rectangular loop as function of frequency for constant horizontal length of one meter with various vertical heights

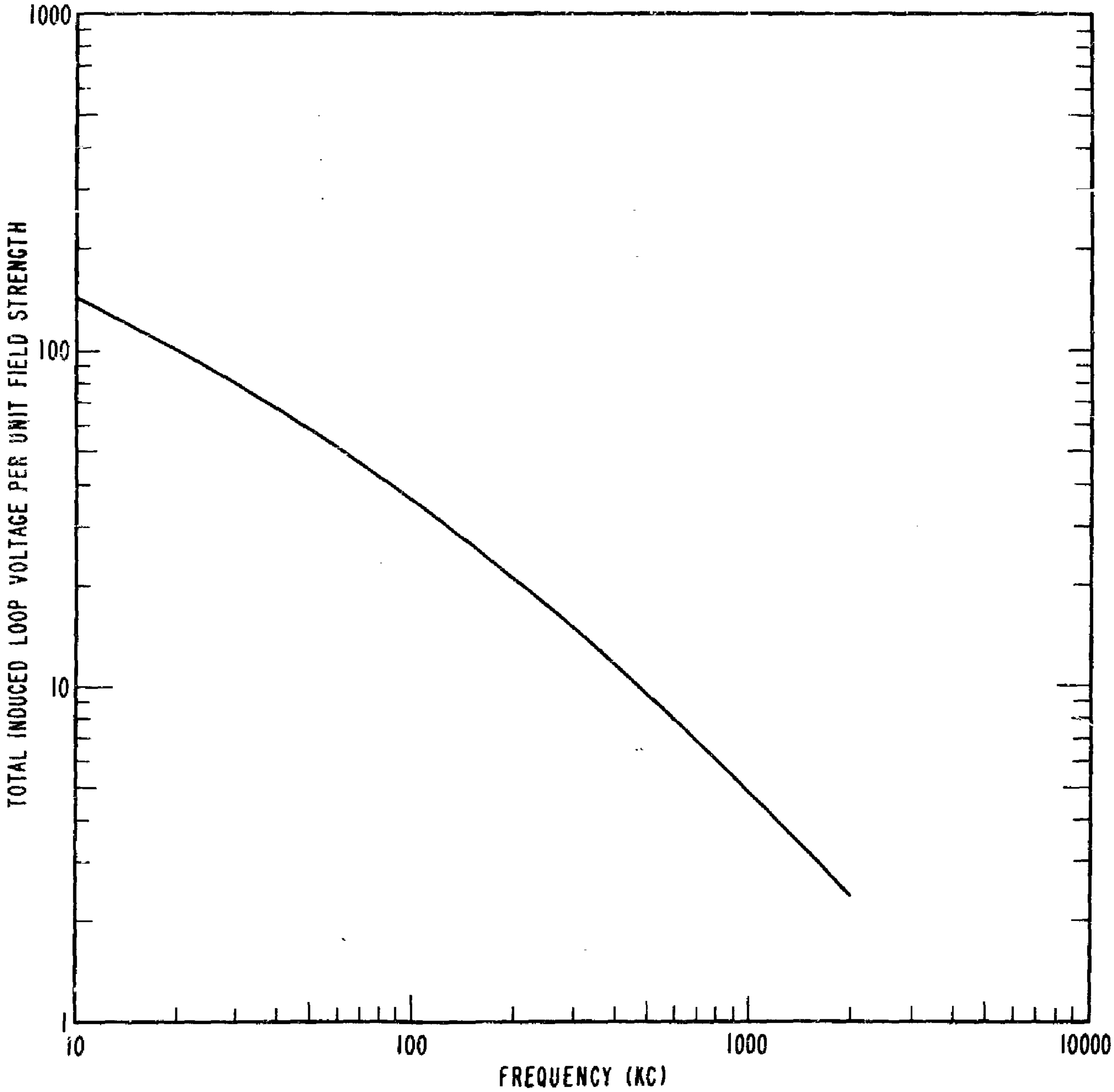

Plate 8 - Total induced loop voltage per unit field strength at top of 2 ft square loop as function of frequency

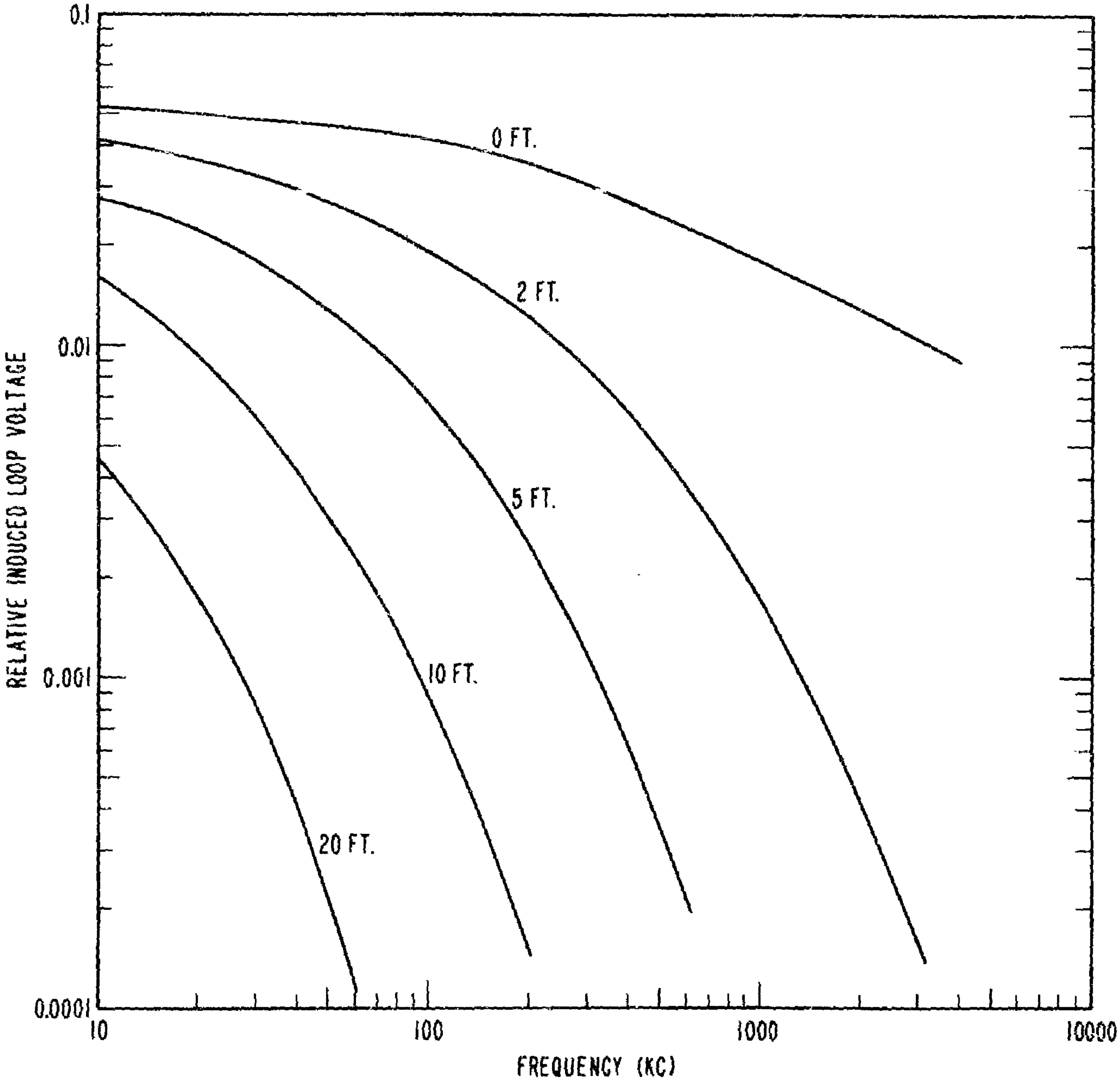

Plate 9 - Total induced loop voltage per unit field strength above surface of water for ground wave as function of frequency for various depths of 2 ft square loop

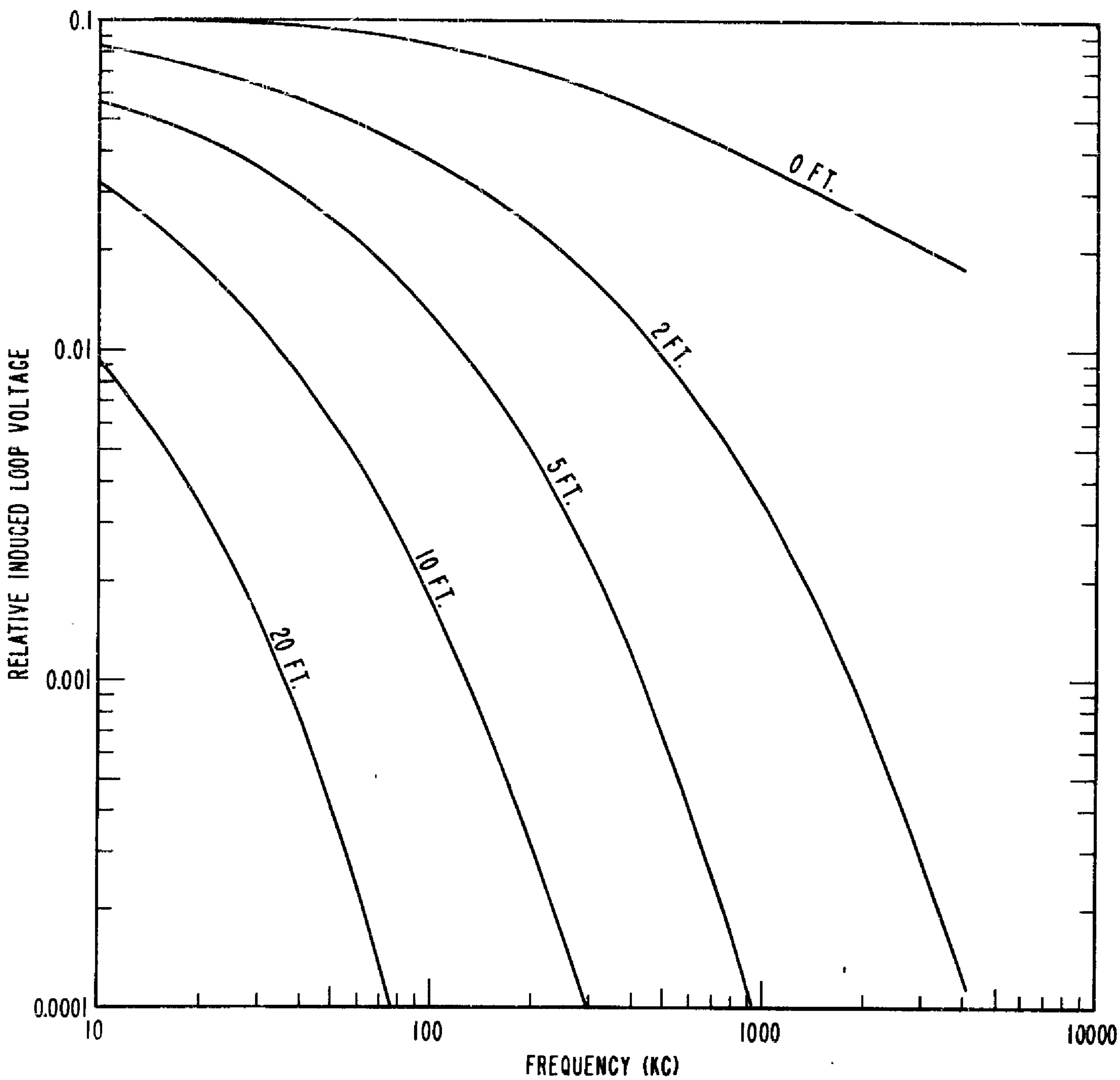

Plate 10 - Total induced loop voltage per unit field strength above surface of water for sky wave as function of frequency for various depths of 2 ft square loop

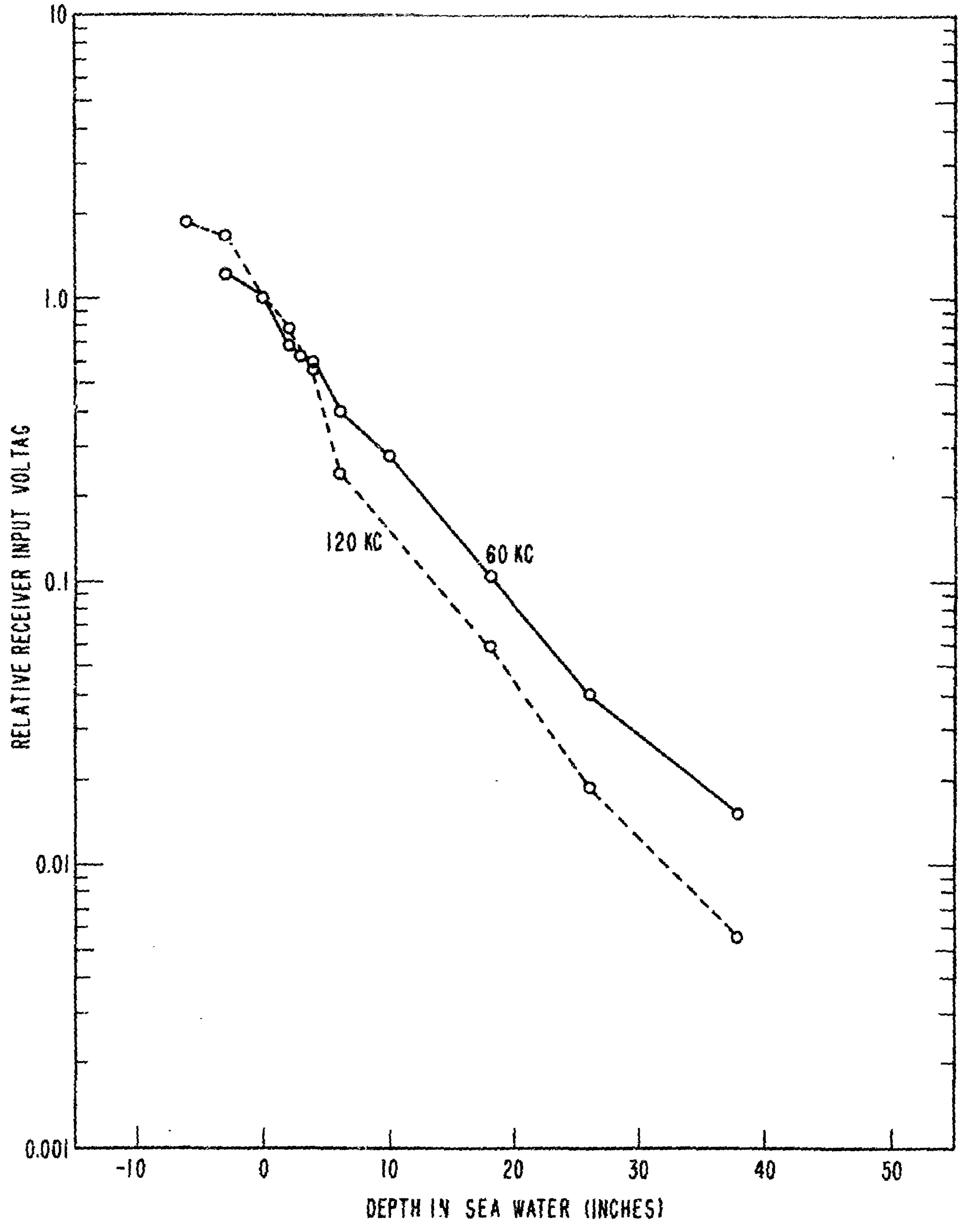

Plate 11 - Relative receiver input voltage as function of depth of center of loop A in tank of sea water

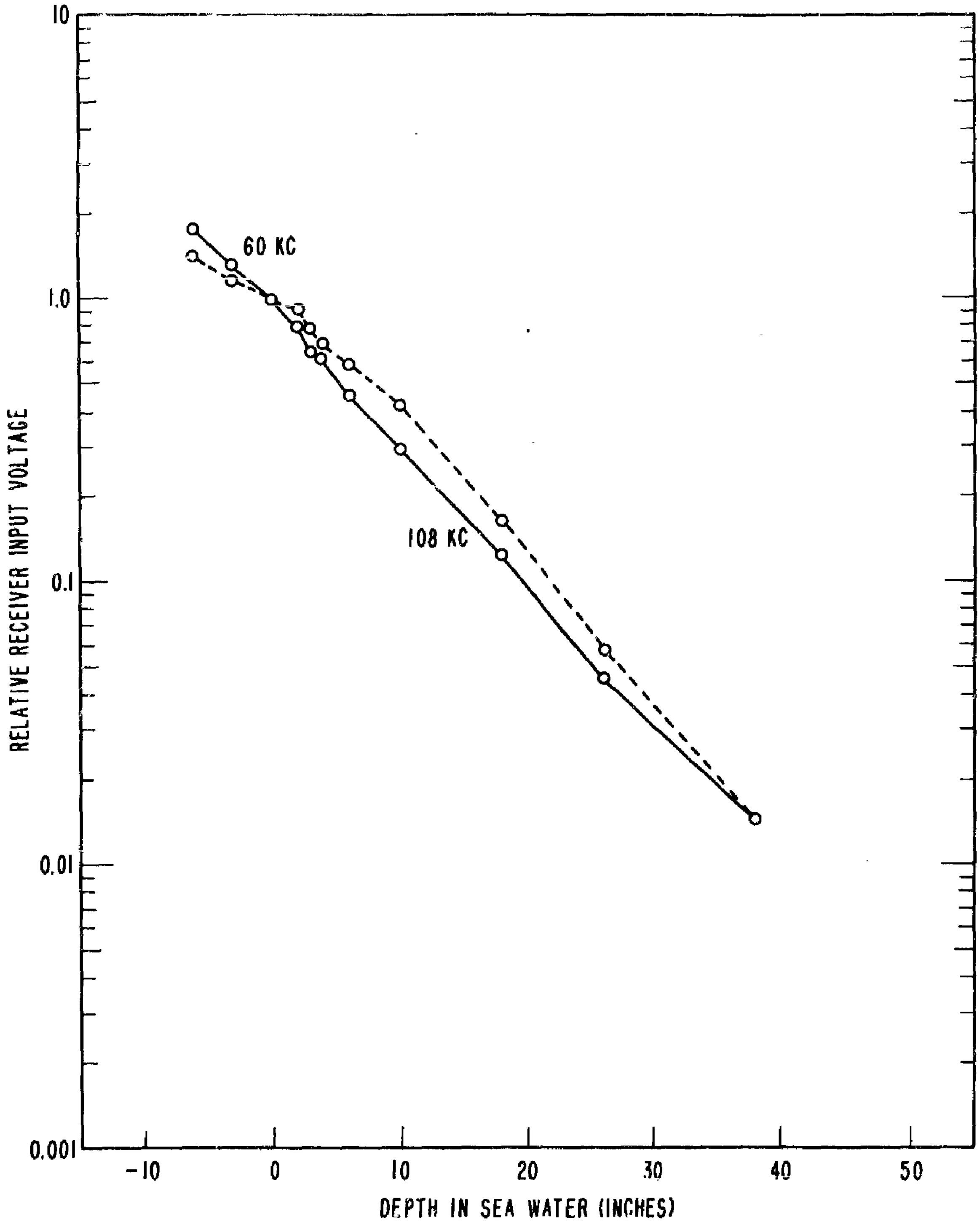

Plate 12 - Relative receiver input voltage as function of depth of center of loop B in tank of sea water

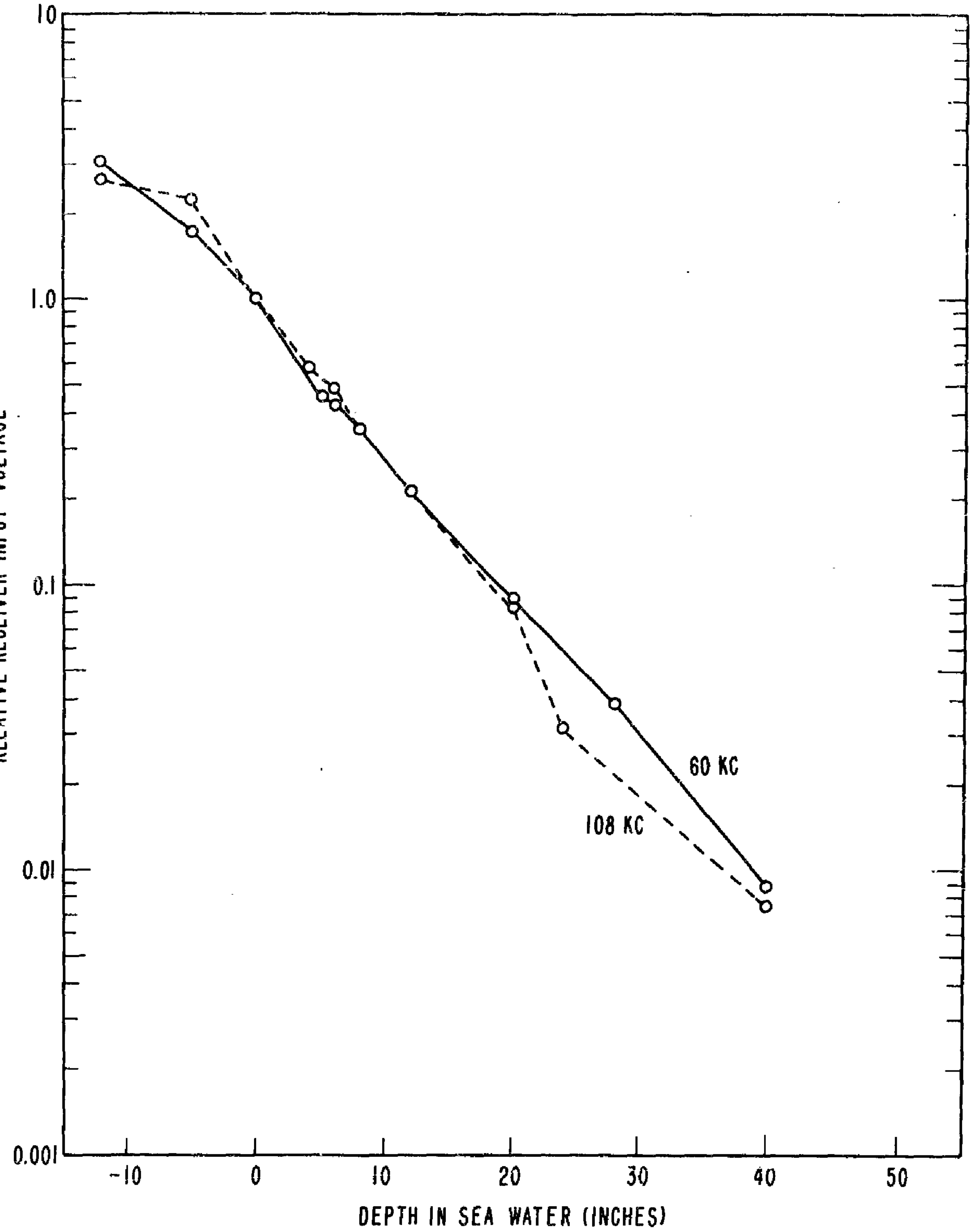

Plate 13 - Relative receiver input voltage as function of depth of center of loop C in tank of sea water

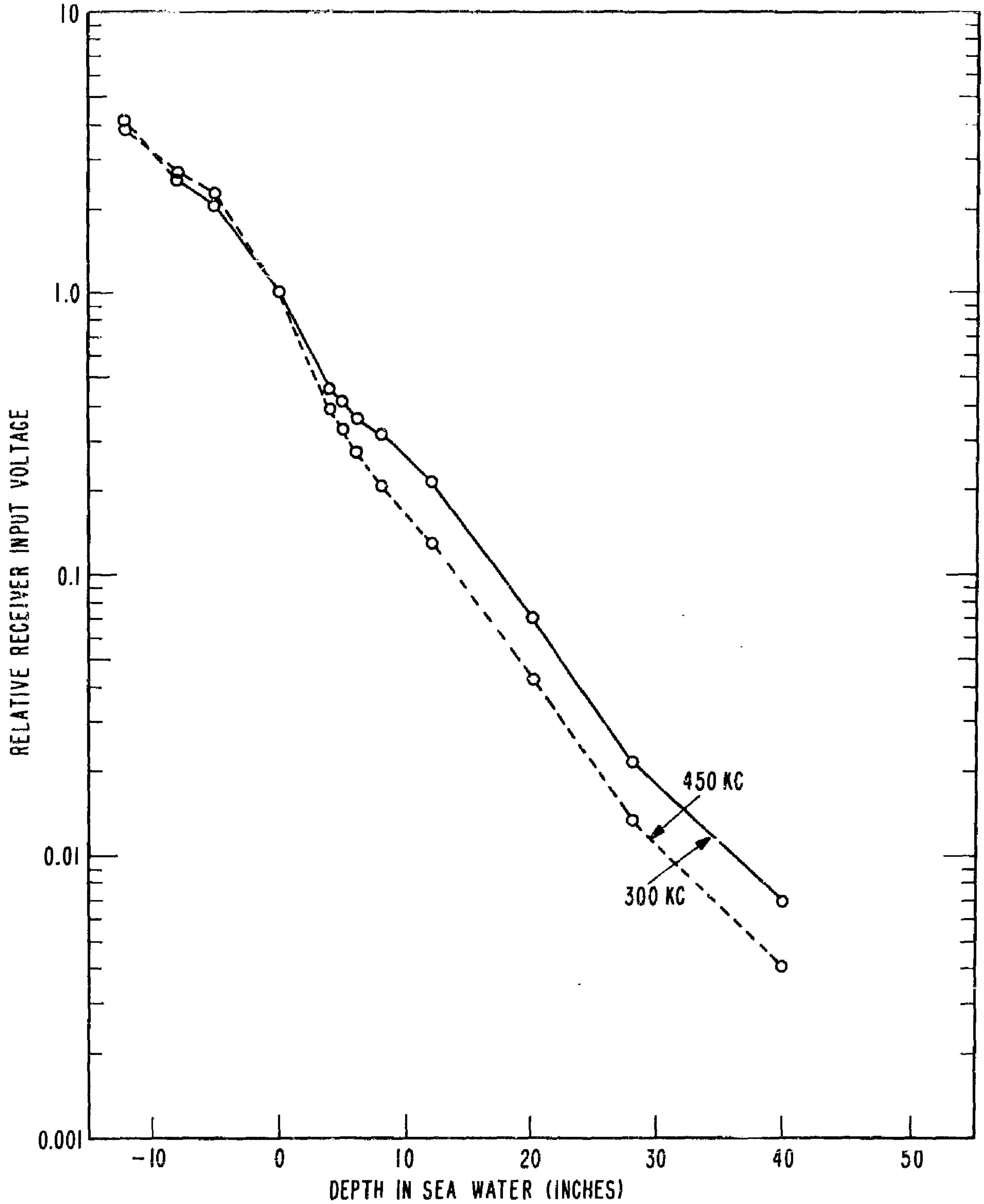

Plate 14 - Relative receiver input voltage as function of depth of center of loop D in tank of sea water

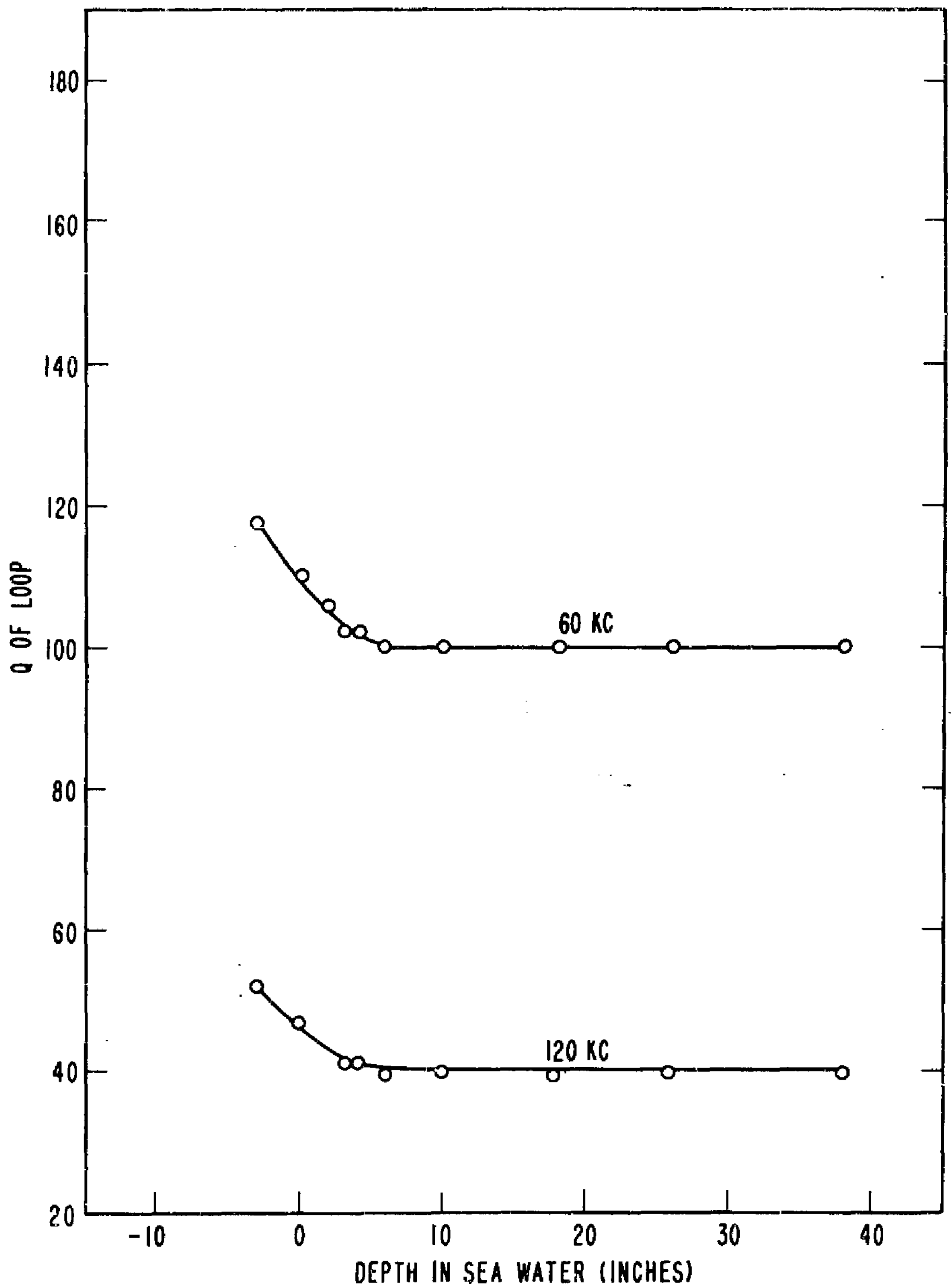

Plate 15 - "Q" as function of depth of center of loop A in tank of sea water

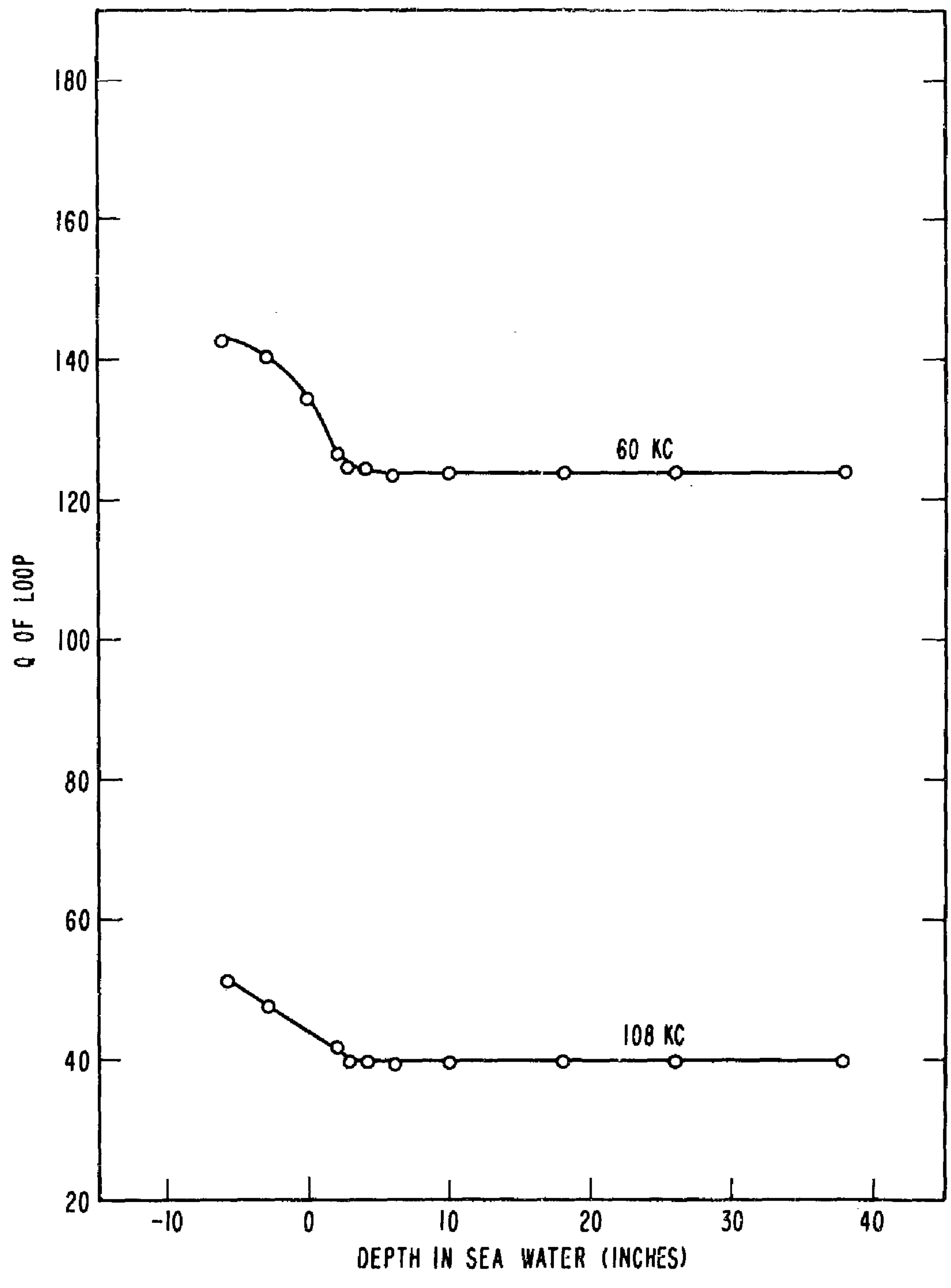

Plate 16 - "Q" as function of depth of center of loop B in tank of sea water

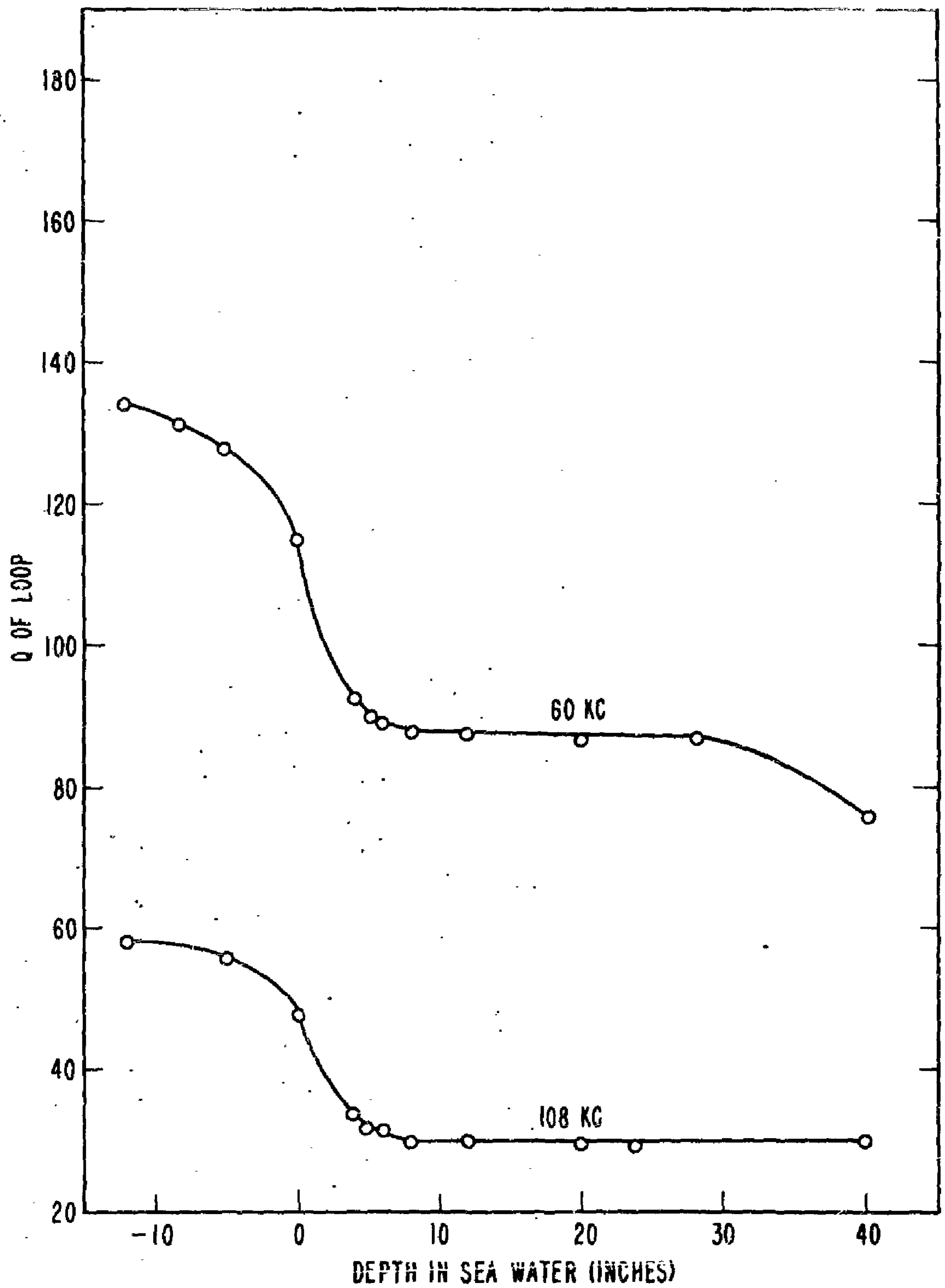

Plate 17 - "Q" as function of depth of center of loop C in tank of sea water

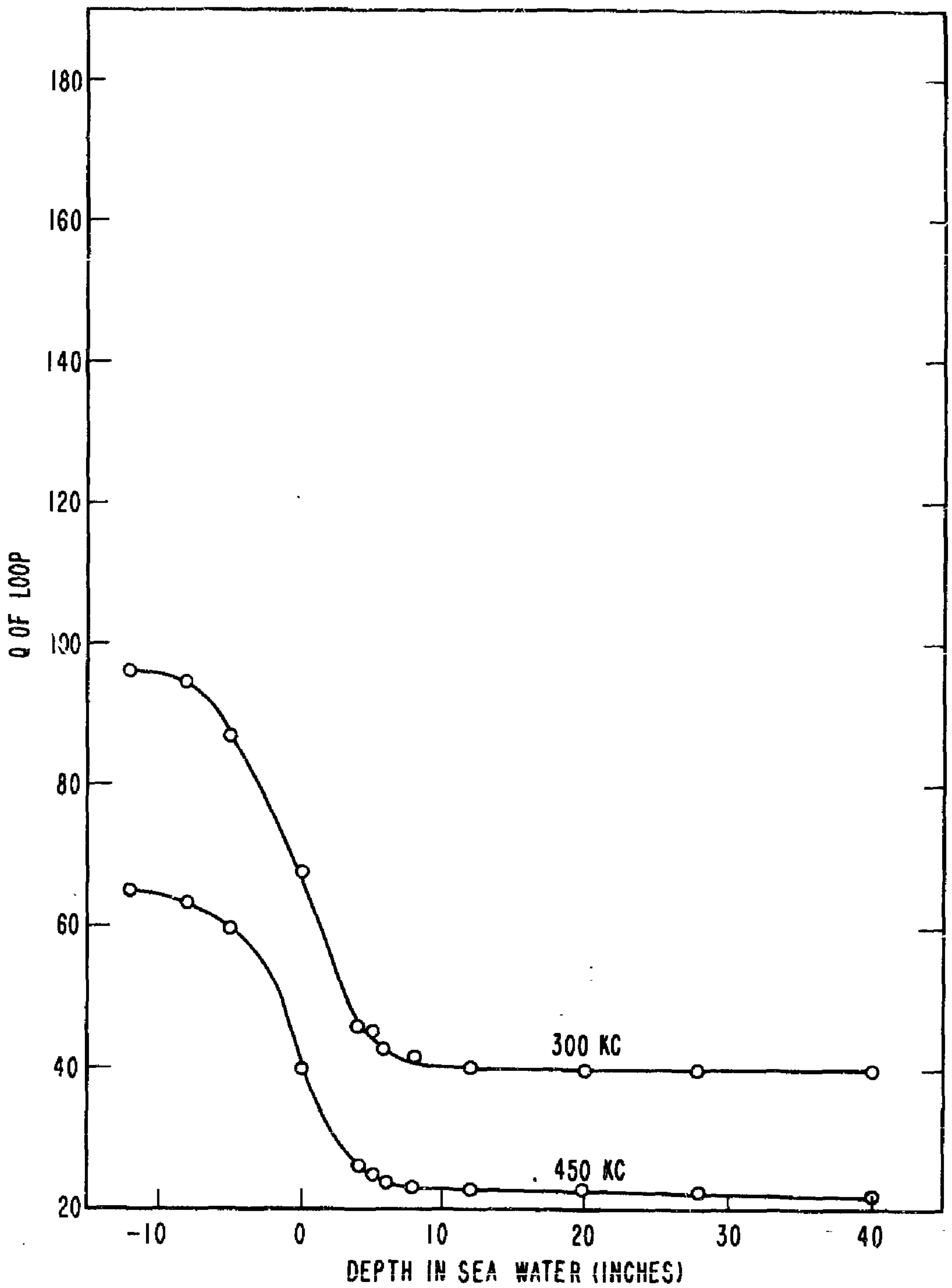

Plate 18 - "Q" as function of depth of center of loop D in tank of sea water

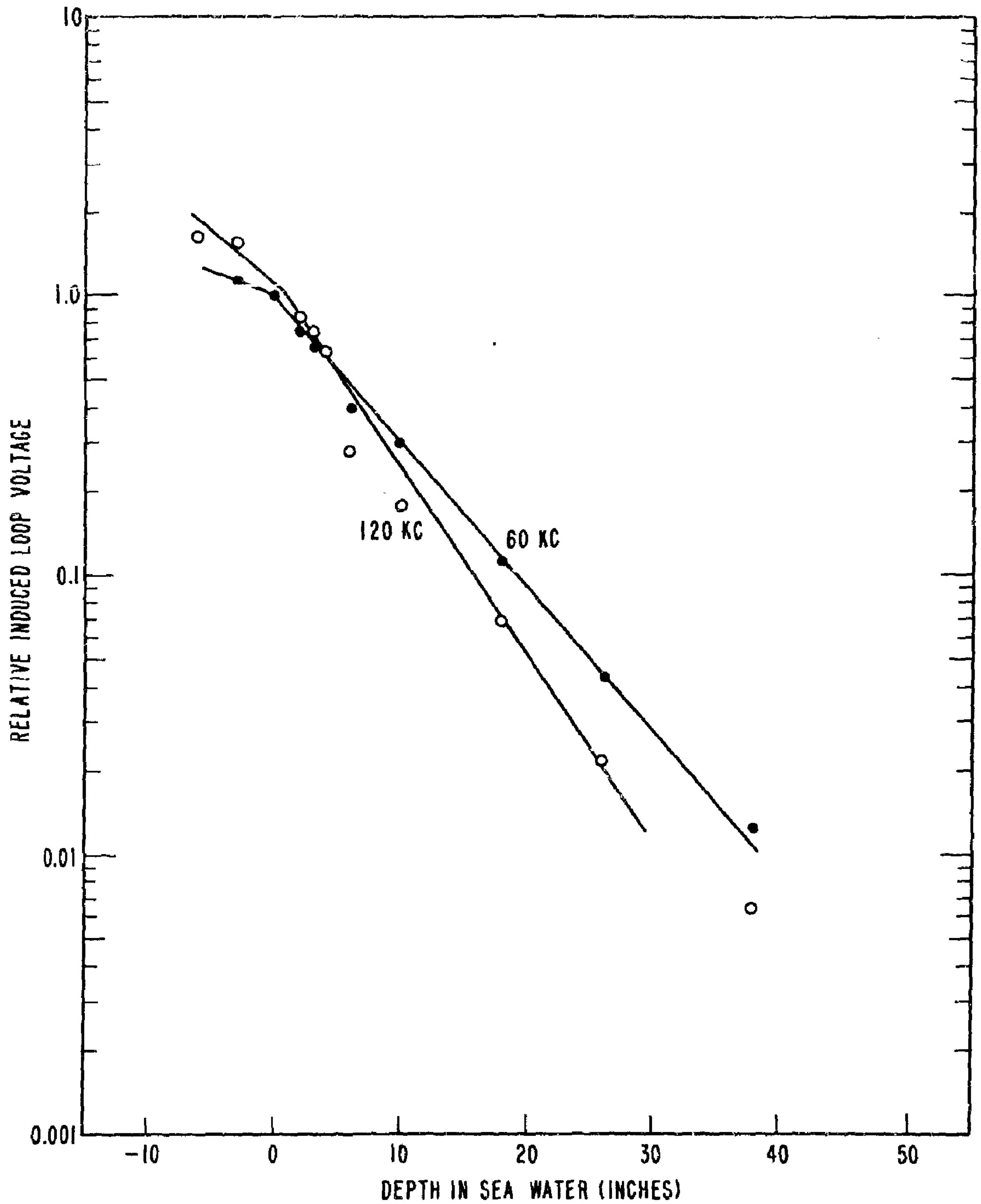

Plate 19 - Total induced loop voltage as function of depth of center of loop A in tank of sea water

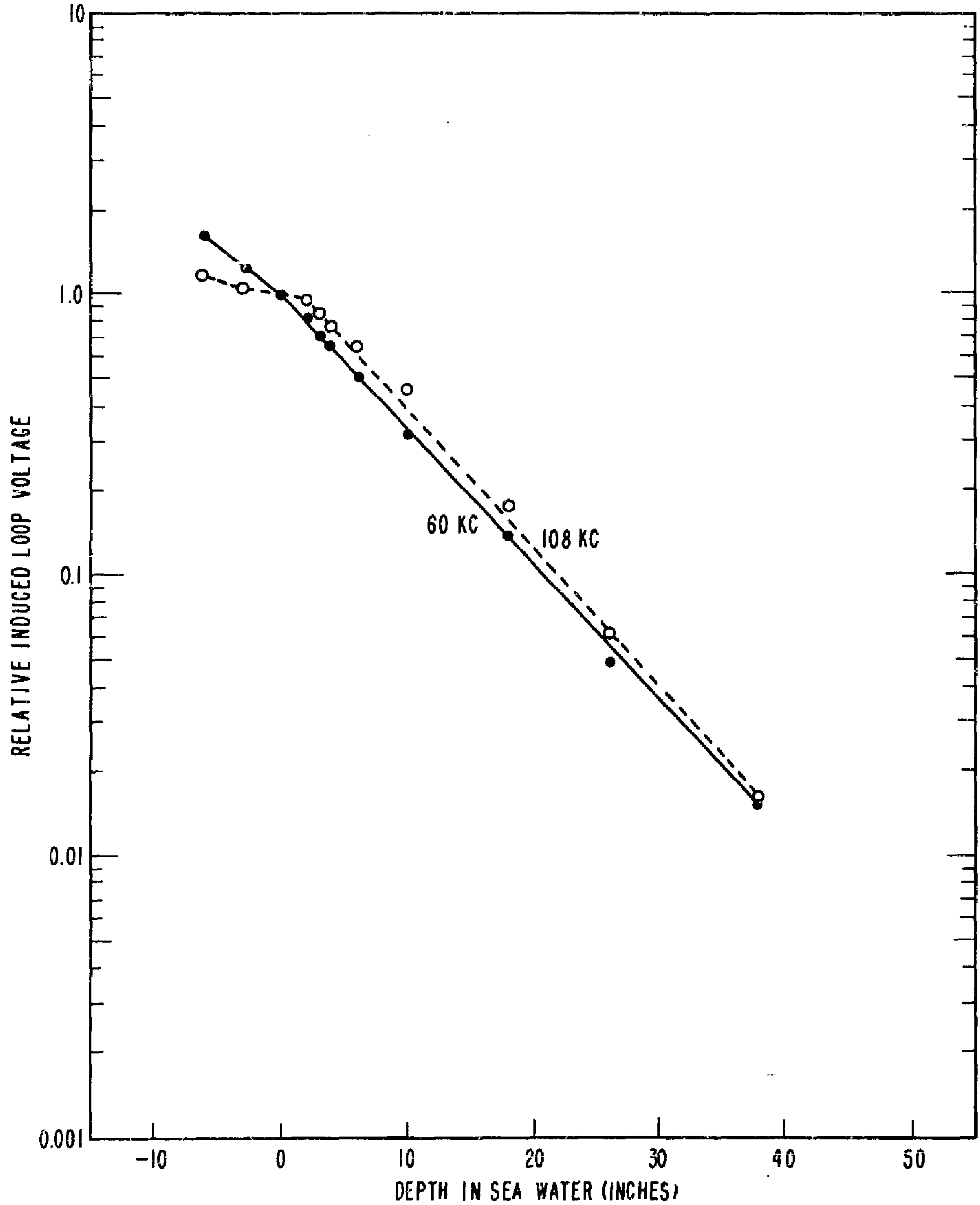

Plate 20 - Total induced loop voltage as function of depth of center of loop B in tank of sea water

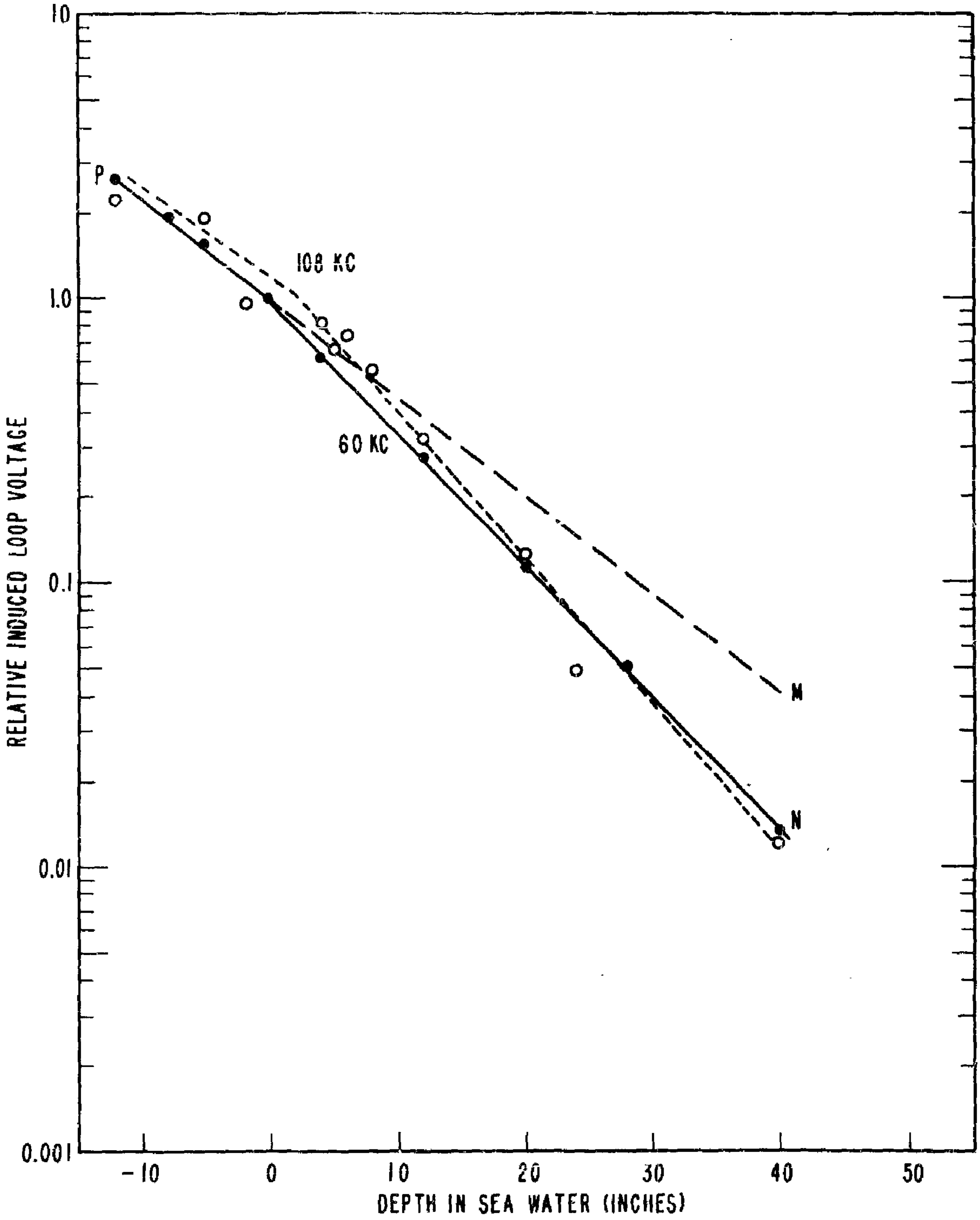

Plate 21 - Total induced loop voltage as function of depth of center of loop C in tank of sea water

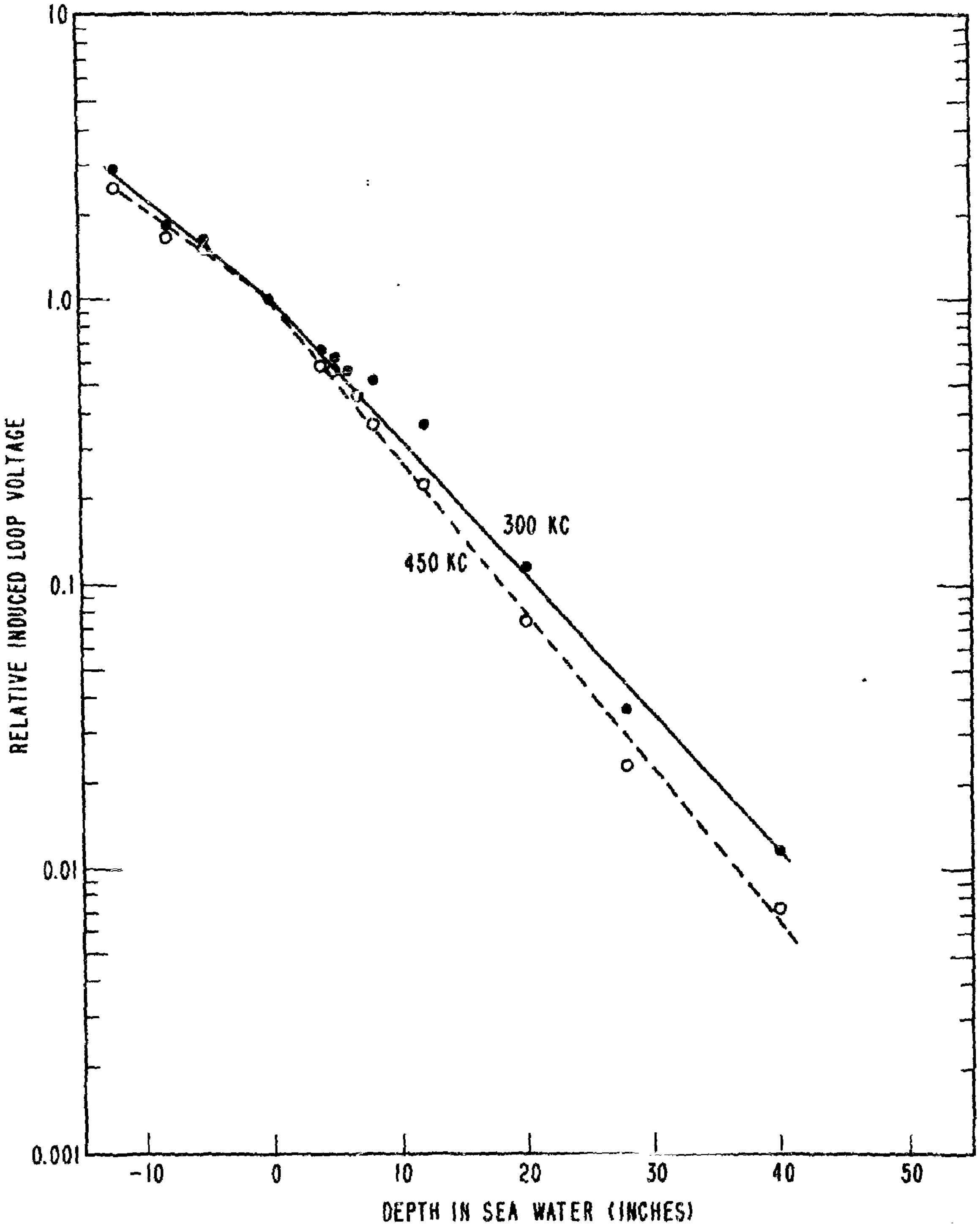

Plate 22 - Total induced loop voltage as function of depth of center of loop D in tank of sea water

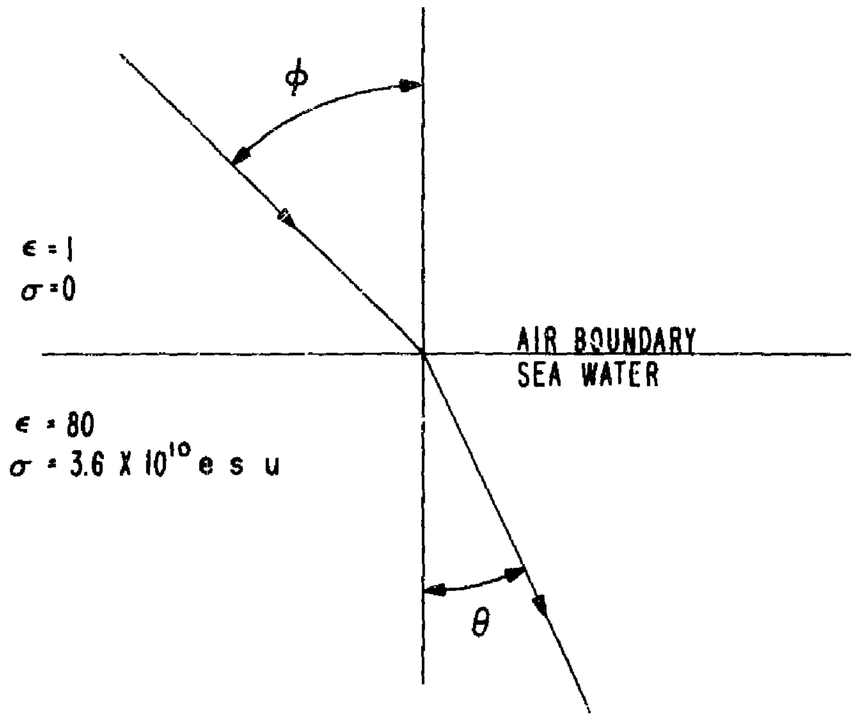

Figure 1

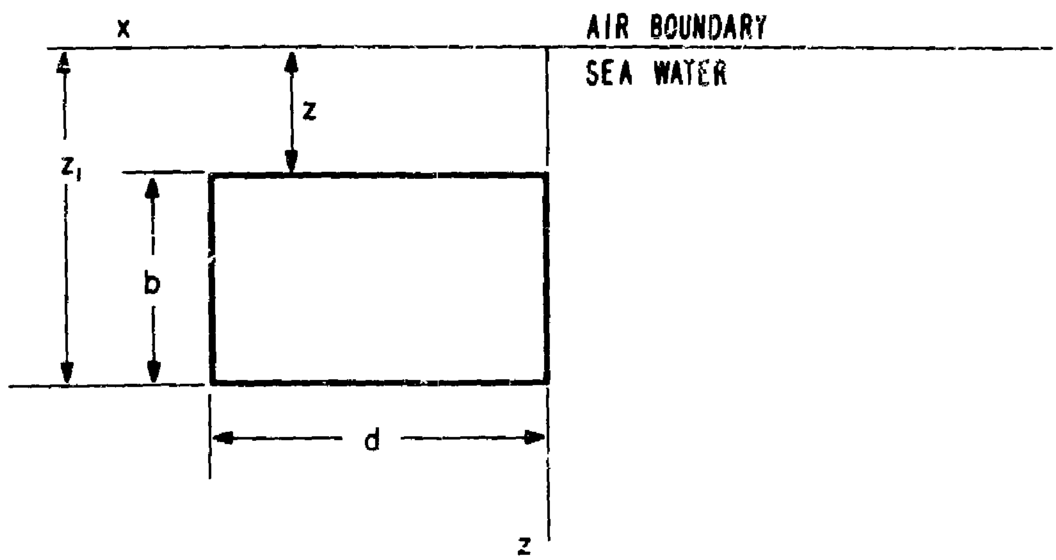

Figure 2

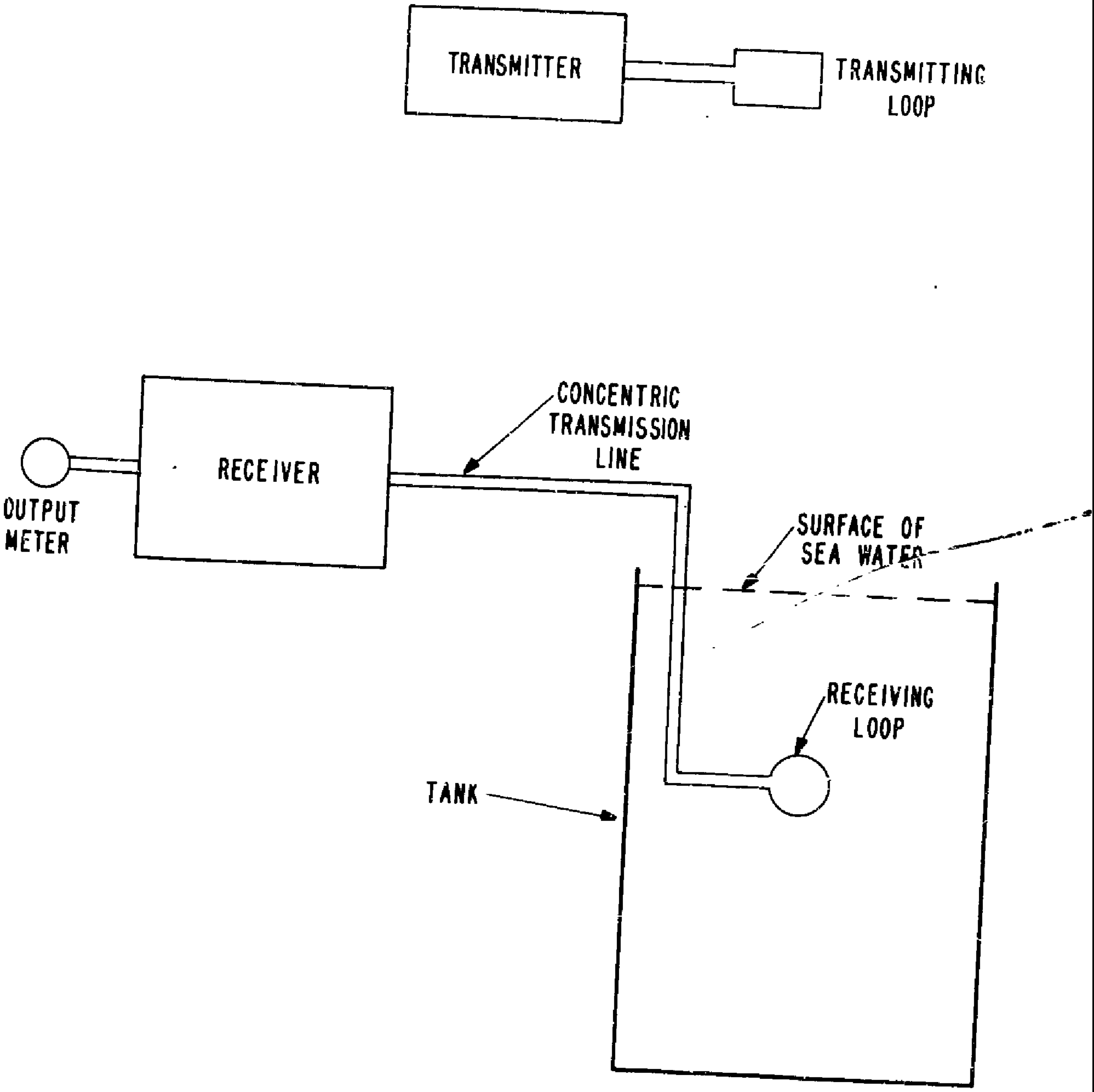
TRANSMITTER
TRANSMITTING LOOP
OUTPUT METER
RECEIVER
CONCENTRIC TRANSMISSION LINE
SURFACE OF SEA WATER
RECEIVING LOOP
TANK

Figure 3

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